



STRUCTURAL AGNOSTIC SPMV: ADAPTING CSR-ADAPTIVE FOR IRREGULAR MATRICES

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Demonstrate how to save space and time by using the CSR format for GPU-based SpMV

Dynamic GPU-based SpMV algorithm that is efficient for regular and irregular matrices

2x faster than CSR-Adaptive
36% faster than CSR5 at 10x less overheads

SPARSE MATRIX-VECTOR MULTIPLICATION (SPMV)



1.0	-	2.0	-	3.0
-	4.0	-	5.0	-
-	-	6.0	-	-
7.0	-	-	8.0	-
-	9.0	-	-	-

 $\times$

1
2
3
4
5

 $=$

SPARSE MATRIX-VECTOR MULTIPLICATION (SPMV)



1.0	-	2.0	-	3.0
-	4.0	-	5.0	-
-	-	6.0	-	-
7.0	-	-	8.0	-
-	9.0	-	-	-

1×1

$\times$

1
2
3
4
5

$=$

SPARSE MATRIX-VECTOR MULTIPLICATION (SPMV)



1.0	-	2.0	-	3.0
-	4.0	-	5.0	-
-	-	6.0	-	-
7.0	-	-	8.0	-
-	9.0	-	-	-

$1*1 + 2*3$

$\times$

1
2
3
4
5

$=$

SPARSE MATRIX-VECTOR MULTIPLICATION (SPMV)



1.0	-	2.0	-	3.0
-	4.0	-	5.0	-
-	-	6.0	-	-
7.0	-	-	8.0	-
-	9.0	-	-	-

$1*1 + 2*3 + 3*5$

$\times$

1
2
3
4
5

$=$

SPARSE MATRIX-VECTOR MULTIPLICATION (SPMV)



1.0	-	2.0	-	3.0
-	4.0	-	5.0	-
-	-	6.0	-	-
7.0	-	-	8.0	-
-	9.0	-	-	-

$1*1 + 2*3 + 3*5$

$\times$

1
2
3
4
5

$=$

22

SPARSE MATRIX-VECTOR MULTIPLICATION (SPMV)



1.0	-	2.0	-	3.0
-	4.0	-	5.0	-
-	-	6.0	-	-
7.0	-	-	8.0	-
-	9.0	-	-	-

 $\times$

1
2
3
4
5

 $=$

22
28
18
39
18

SPARSE MATRIX-VECTOR MULTIPLICATION (SPMV)



1.0	-	2.0	-	3.0
-	4.0	-	5.0	-
-	-	6.0	-	-
7.0	-	-	8.0	-
-	9.0	-	-	-

 $\times$

1
2
3
4
5

 $=$

22
28
18
39
18

Traditionally Bandwidth Limited

Performance of SpMV is dominated by
how
the sparse matrix is stored in memory

COMPRESSED SPARSE ROW (CSR)



1.0	-	2.0	-	3.0
-	4.0	-	5.0	-
-	-	6.0	-	-
7.0	-	-	8.0	-
-	9.0	-	-	-

values:

--	--	--	--	--	--	--	--	--

COMPRESSED SPARSE ROW (CSR)



1.0	-	2.0	-	3.0
-	4.0	-	5.0	-
-	-	6.0	-	-
7.0	-	-	8.0	-
-	9.0	-	-	-

values:

1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
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COMPRESSED SPARSE ROW (CSR)



1.0	-	2.0	-	3.0
-	4.0	-	5.0	-
-	-	6.0	-	-
7.0	-	-	8.0	-
-	9.0	-	-	-

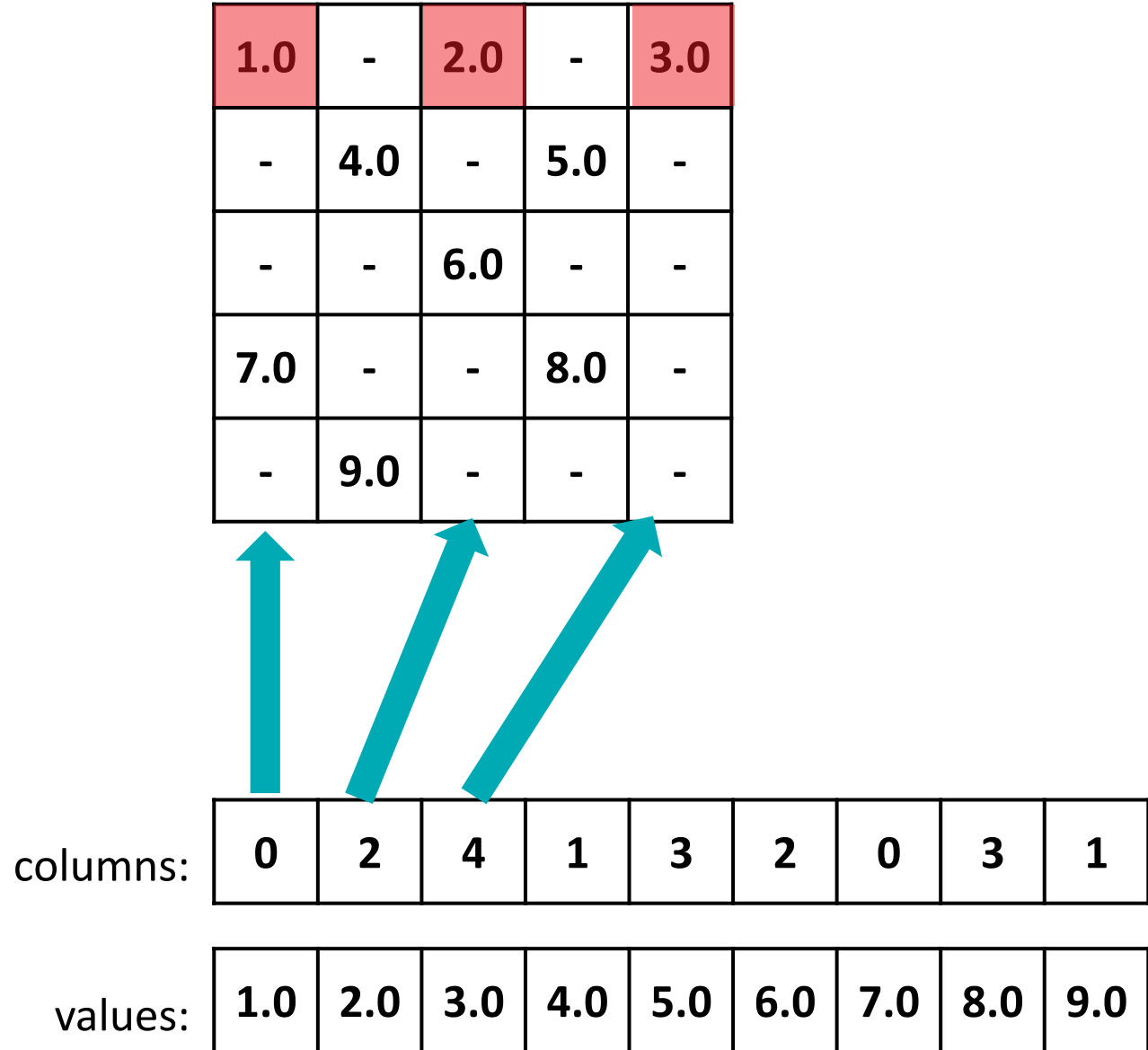
columns:

0	2	4	1	3	2	0	3	1
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values:

1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
------------	------------	------------	------------	------------	------------	------------	------------	------------

COMPRESSED SPARSE ROW (CSR)



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1.0	-	2.0	-	3.0
-	4.0	-	5.0	-
-	-	6.0	-	-
7.0	-	-	8.0	-
-	9.0	-	-	-

row delimiters:

0	3	5	6	8	9
----------	----------	----------	----------	----------	----------

columns:

0	2	4	1	3	2	0	3	1
----------	----------	----------	----------	----------	----------	----------	----------	----------

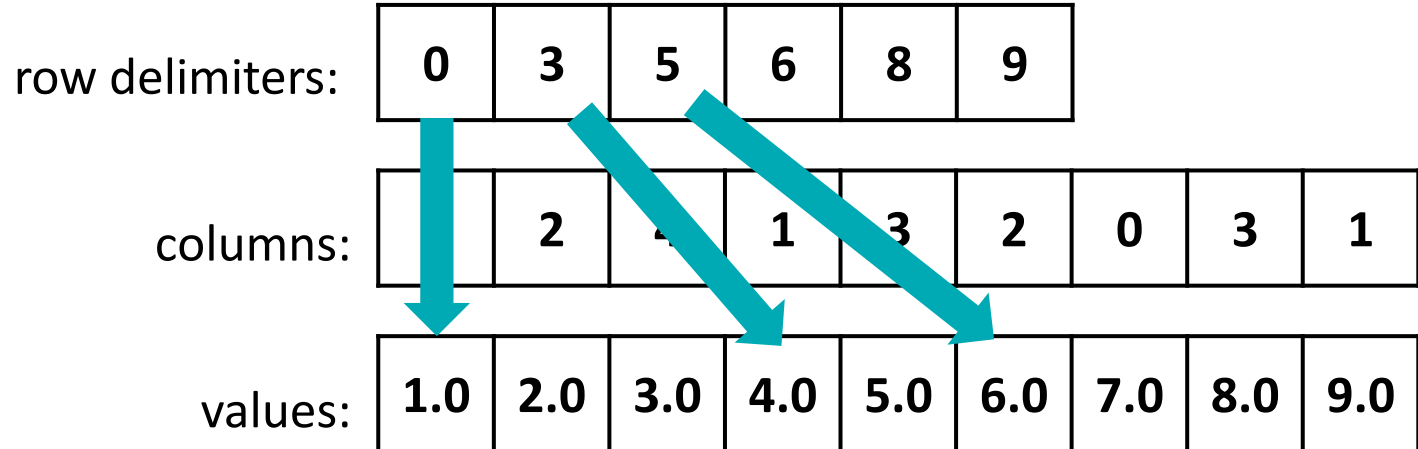
values:

1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
------------	------------	------------	------------	------------	------------	------------	------------	------------

COMPRESSED SPARSE ROW (CSR)



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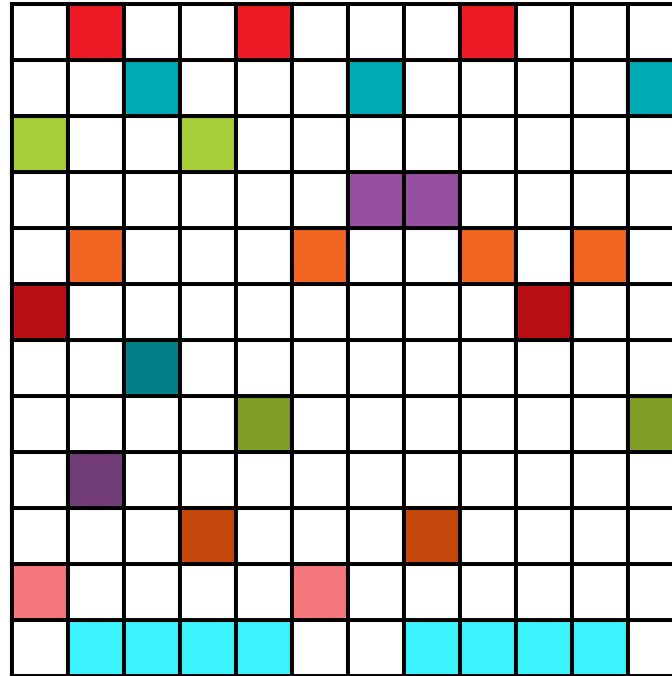
values:

1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
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CSR-SCALAR



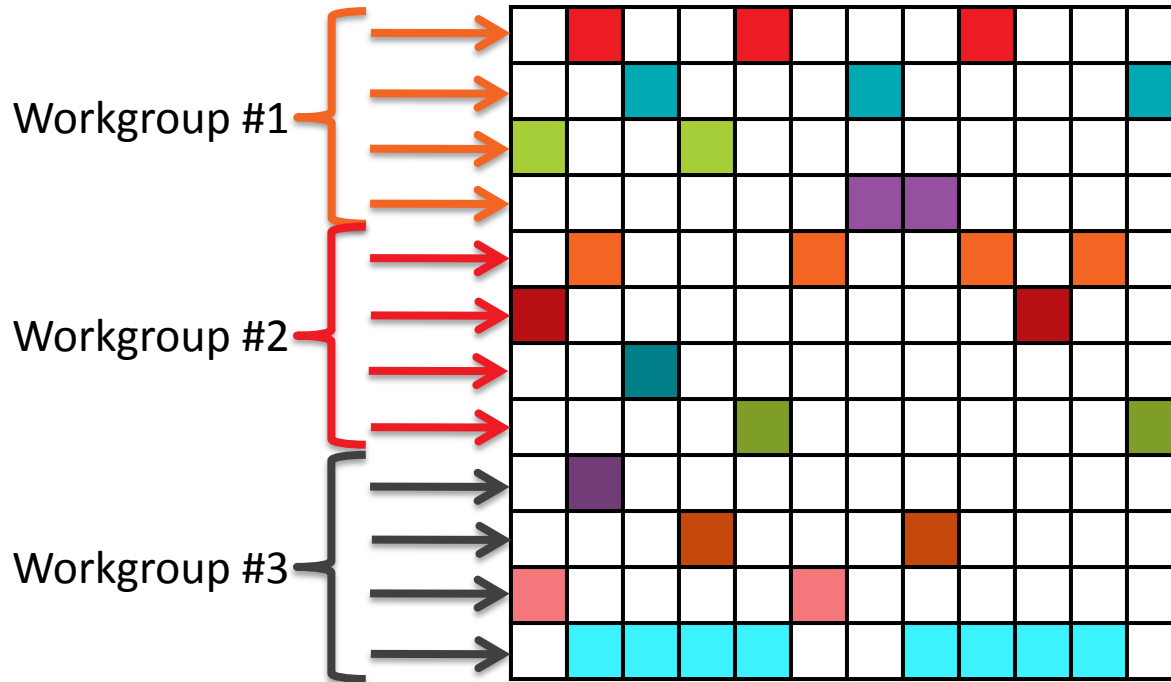
▲ One thread/work-item per matrix row



CSR-SCALAR



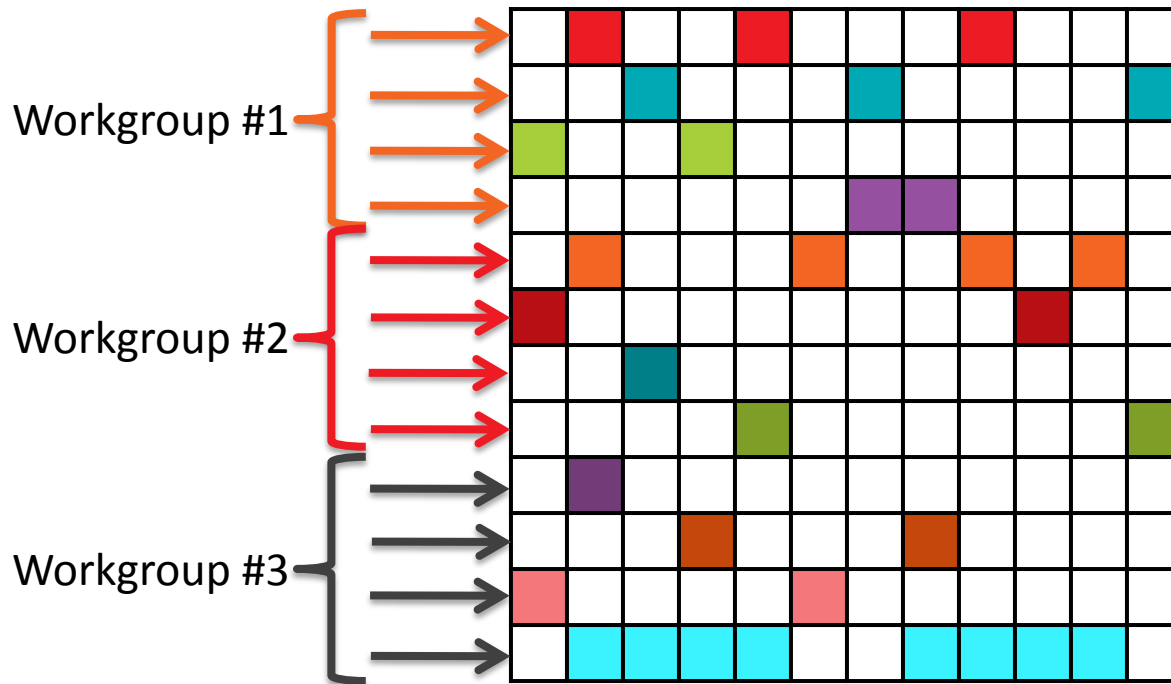
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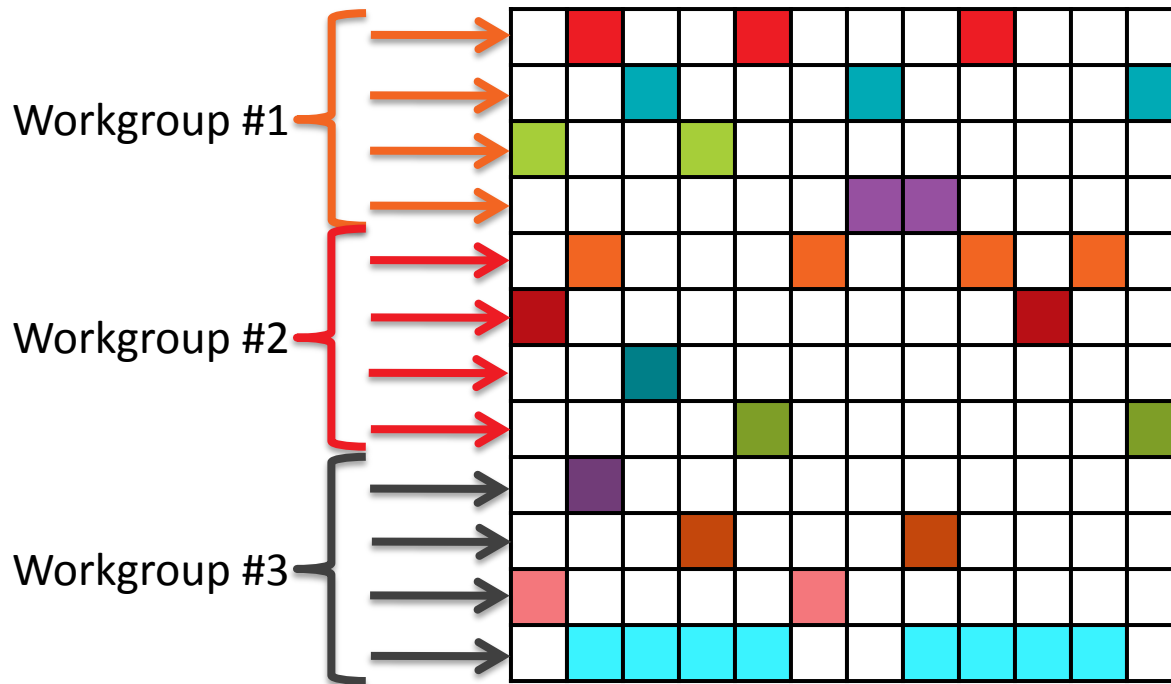
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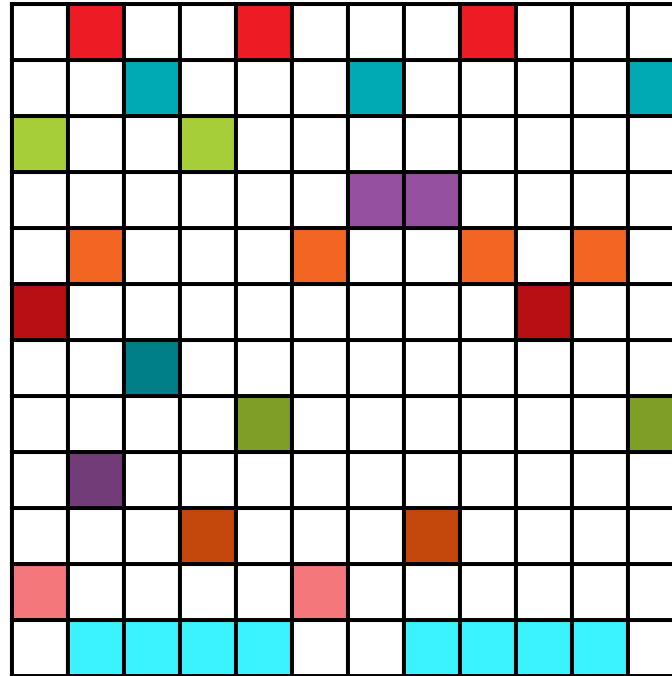
Uncoalesced accesses



CSR-VECTOR



- ▲ One workgroup (multiple threads) per matrix row



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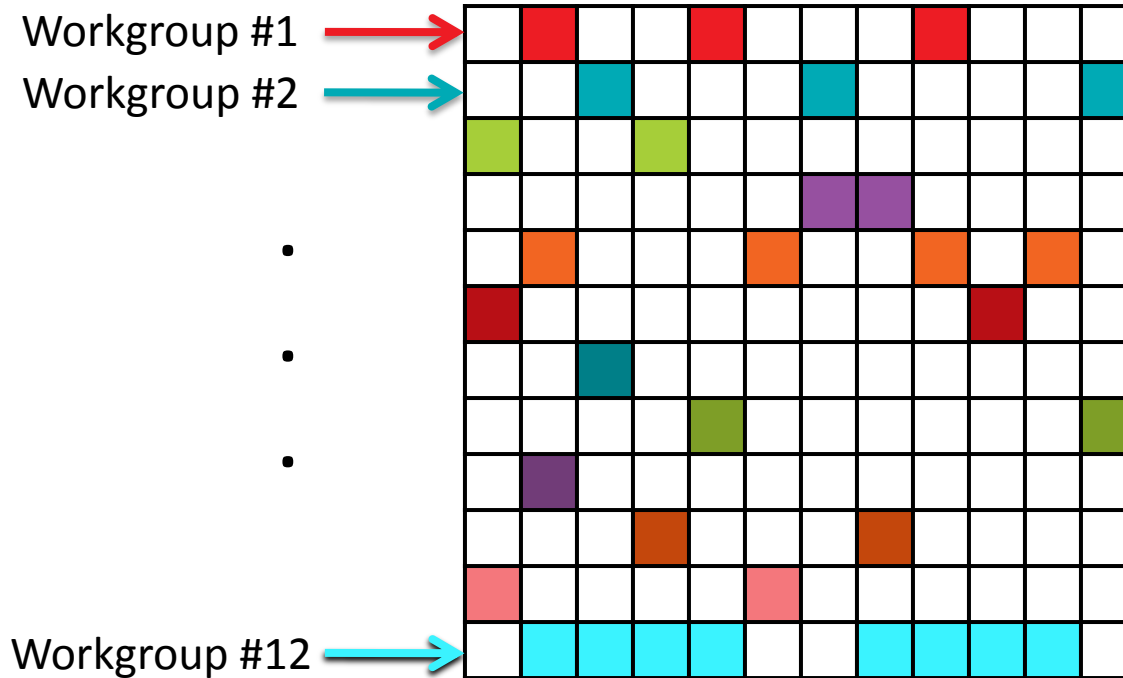
- Workgroup #1 →
- Workgroup #2 →
-
-
-
- Workgroup #12 →
-



CSR-VECTOR



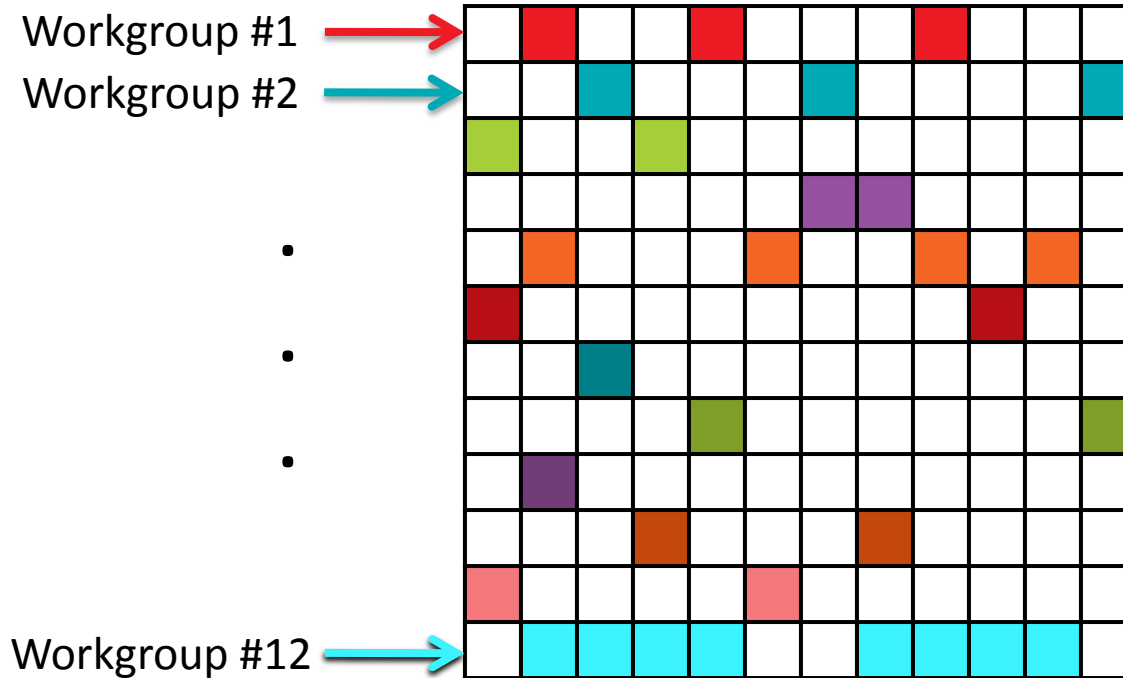
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CSR-VECTOR



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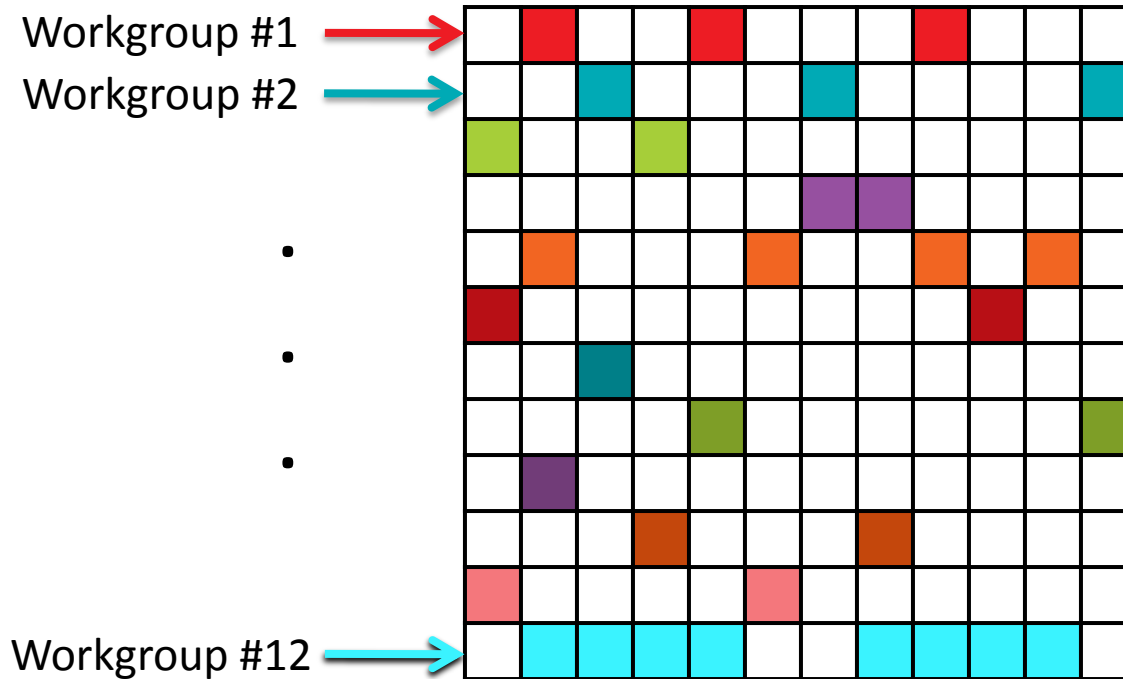
Good: coalesced accesses



CSR-VECTOR



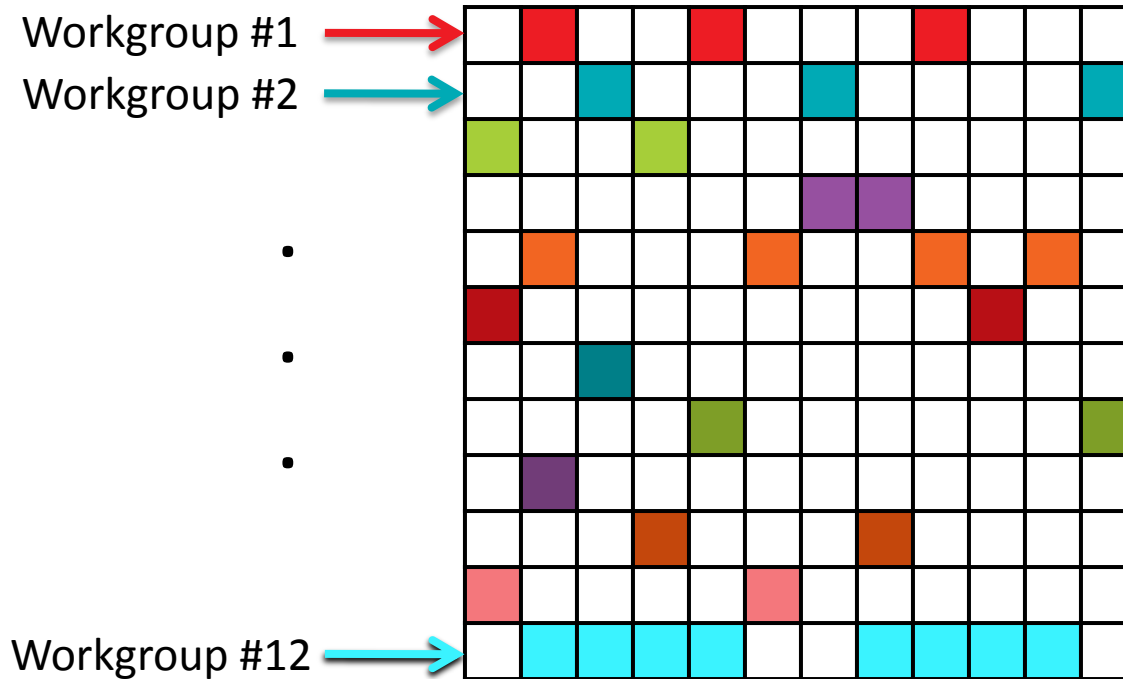
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CSR-VECTOR



▲ One workgroup (multiple threads) per matrix row



Bad: poor parallelism



MAKING SPMV FAST ON GPUS



▲ Neither CSR-scalar nor CSR-vector offer ideal performance

MAKING SPMV FAST ON GPUS



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- ▲ Traditional solution: new matrix storage format + new algorithms
 - More than *fifty* new storage formats have been proposed

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rewrite software

space and time overhead

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- ▲ Traditional solution: new matrix storage format + new algorithms
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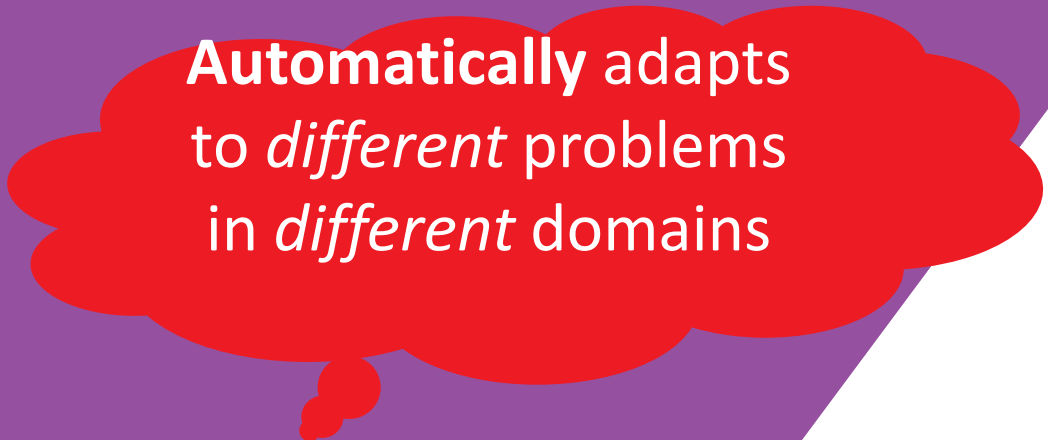
rewrite software

space and time overhead

- ▲ Can we find a good GPU algorithm that does not modify the original CSR data?

The background features a large, solid purple shape that occupies the lower half of the frame. Above this, there are several orange geometric shapes, including a large parallelogram and a smaller, slanted rectangle, creating a layered, abstract effect.

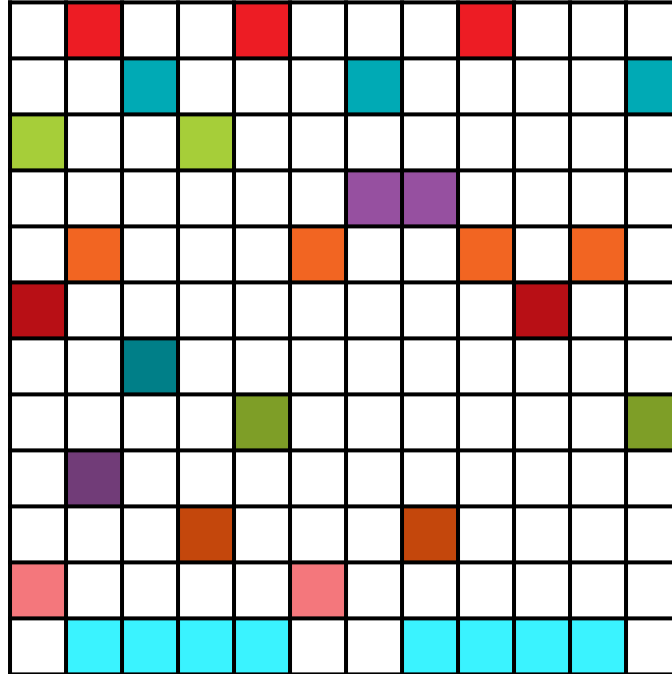
CSR-Adaptive.



Automatically adapts
to *different* problems
in *different* domains

CSR-Adaptive.

INCREASING BANDWIDTH EFFICIENCY



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INCREASING BANDWIDTH EFFICIENCY



▲ How can we get good bandwidth for other “short” rows?



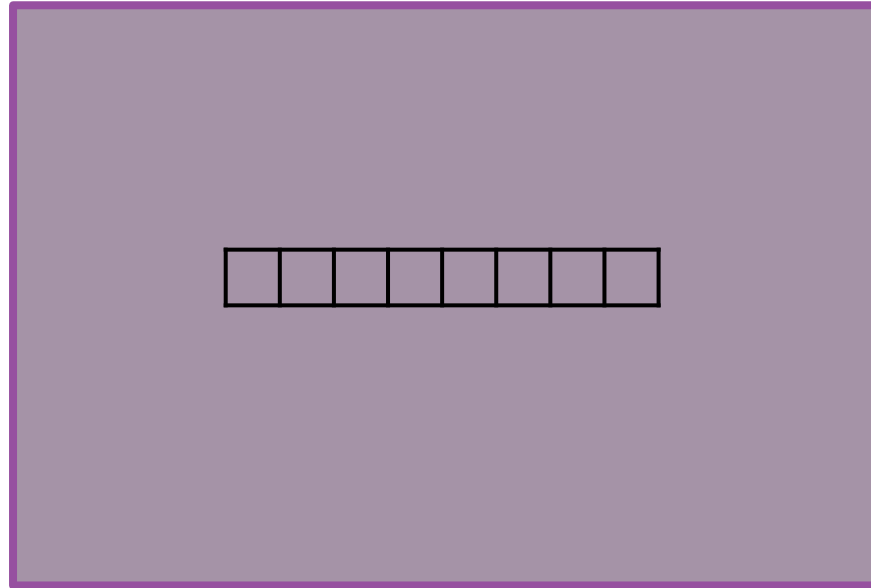
▲ How can we get good bandwidth for other “short” rows?



CSR-STREAM: STREAM MATRIX INTO THE LDS



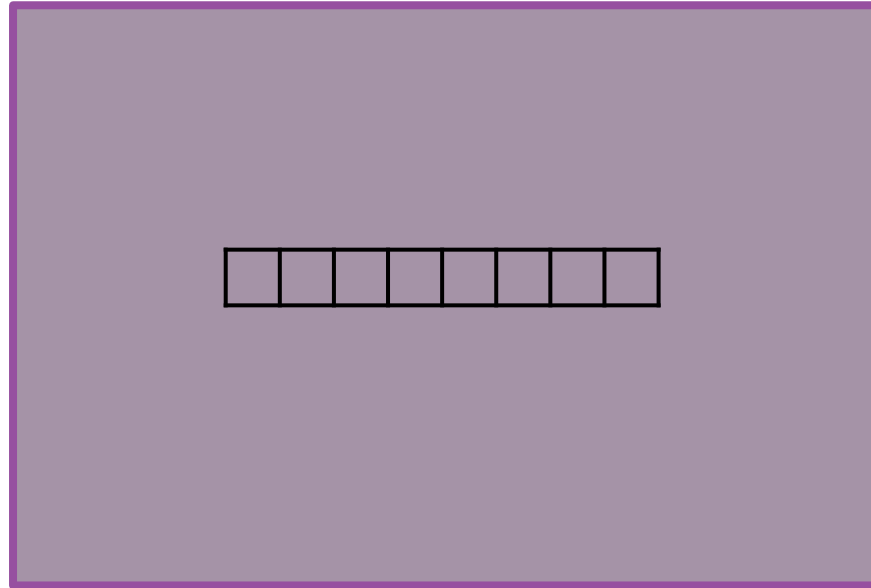
Local Data Share



CSR-STREAM: STREAM MATRIX INTO THE LDS



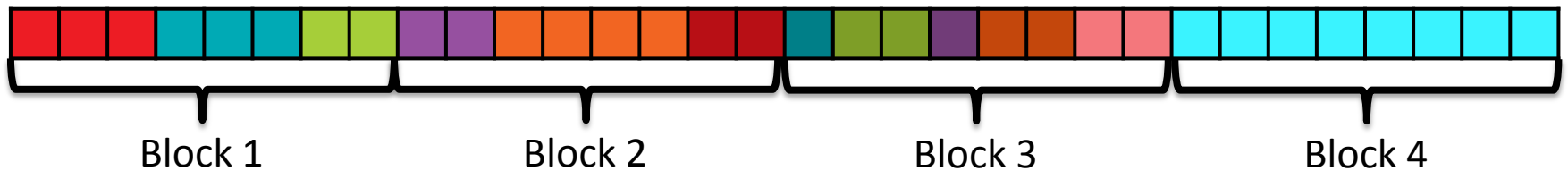
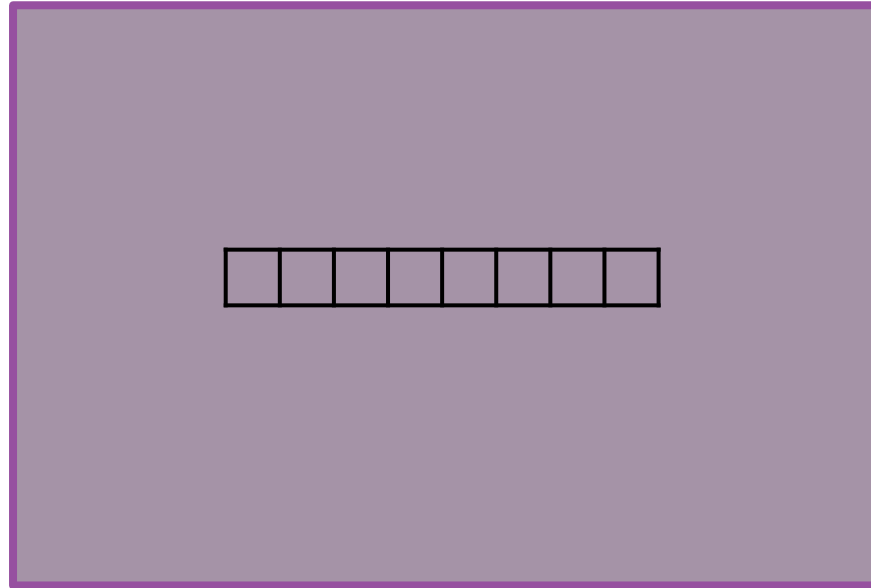
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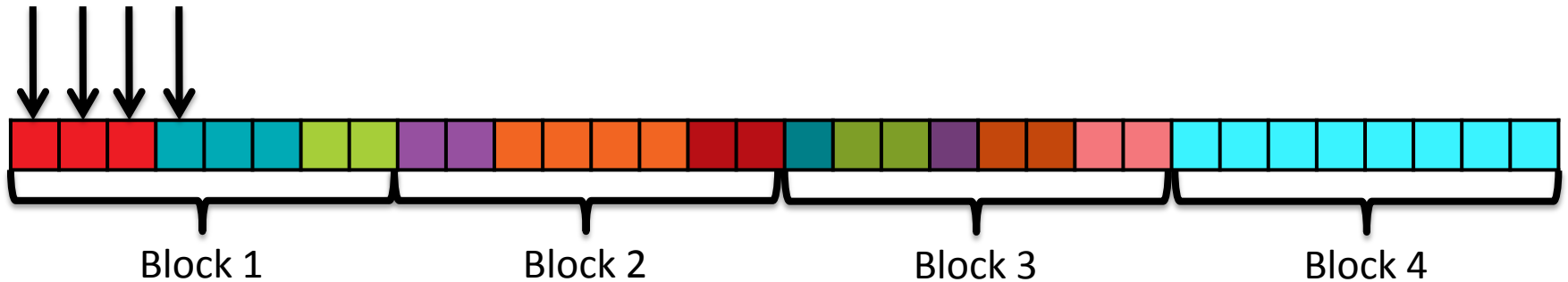
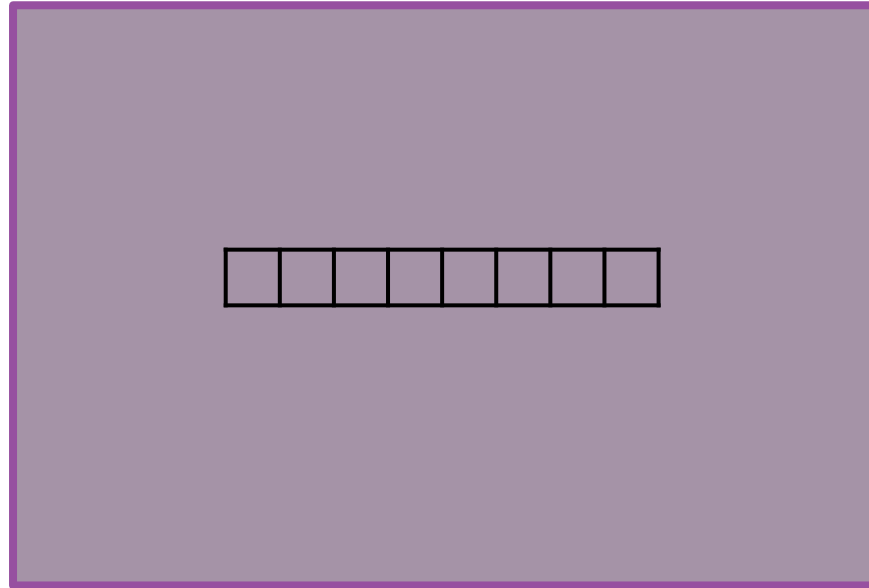
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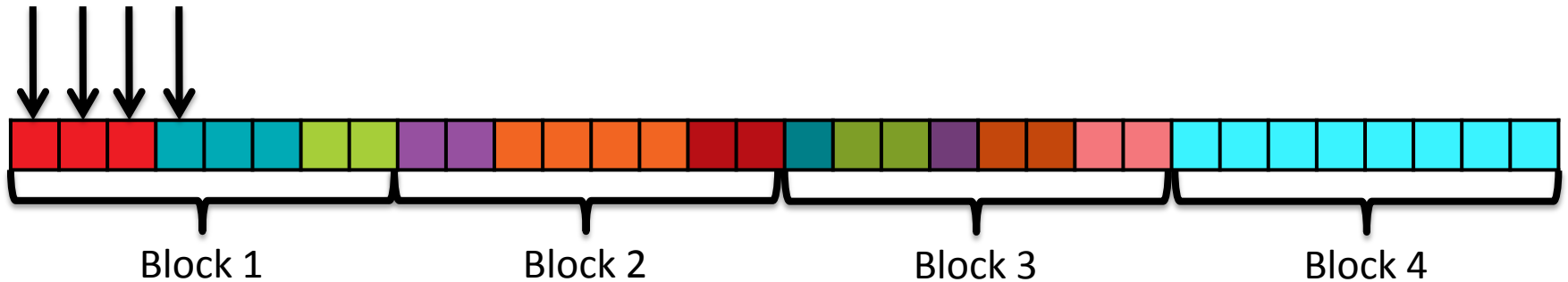
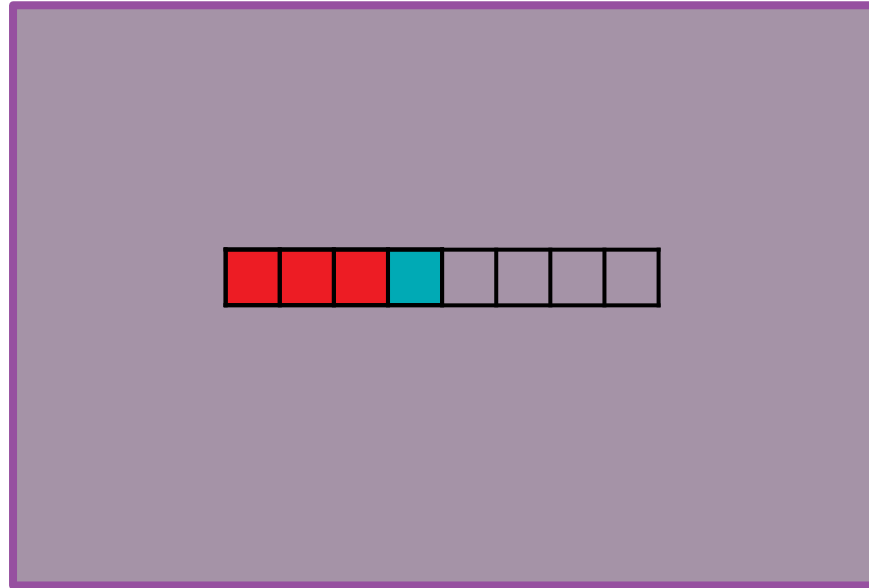
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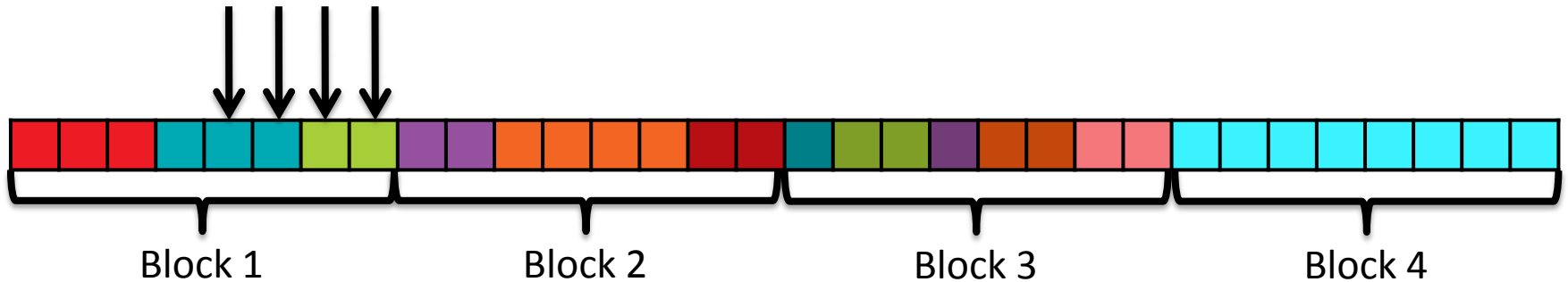
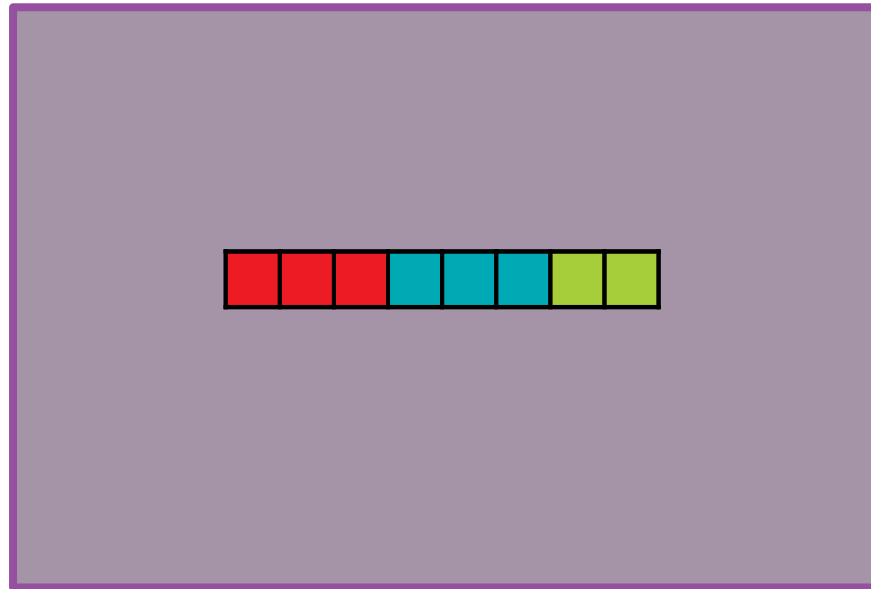
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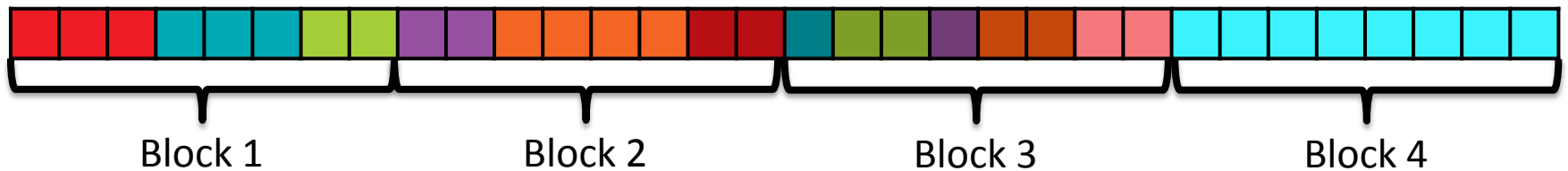
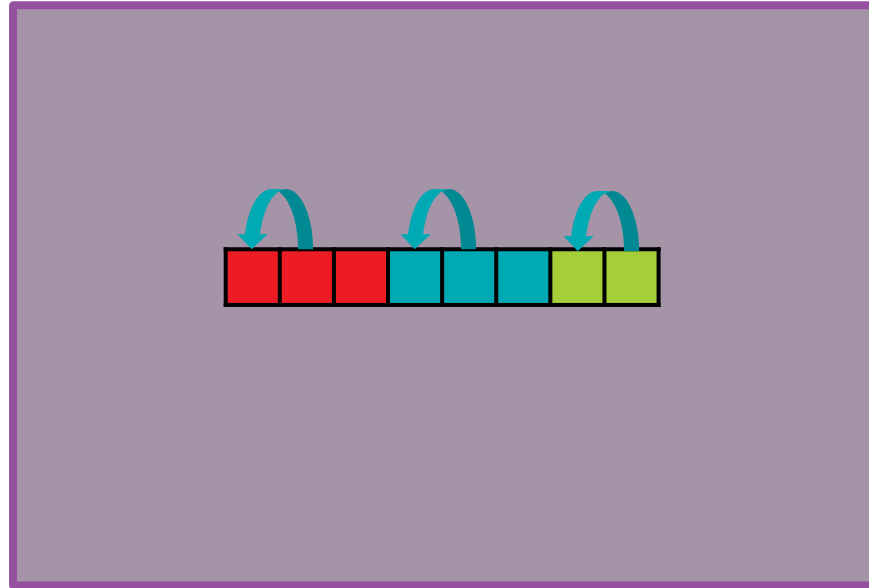
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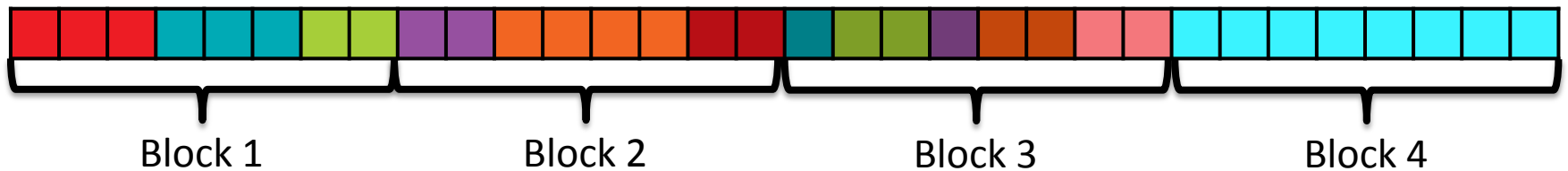
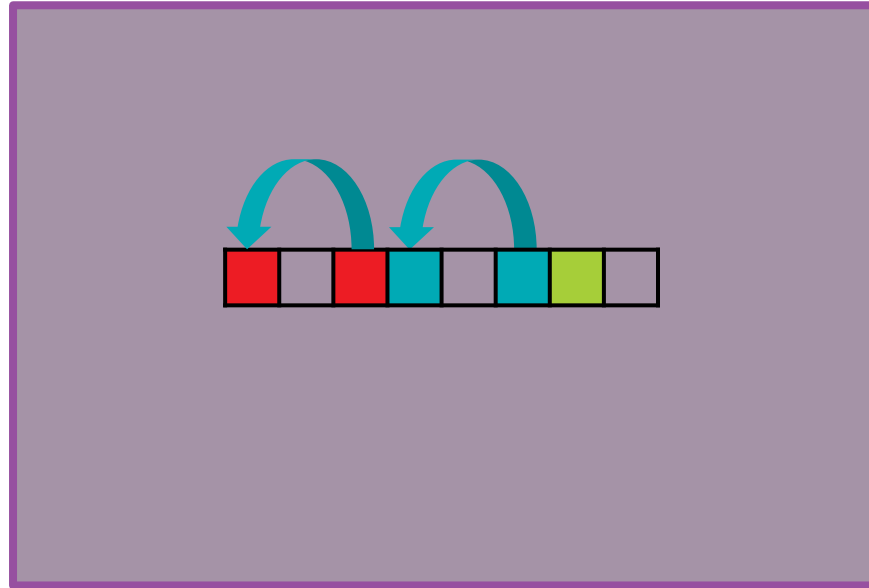
Local Data Share



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Local Data Share



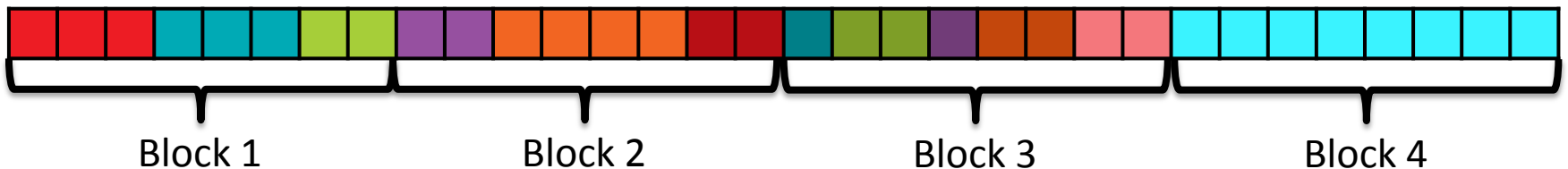
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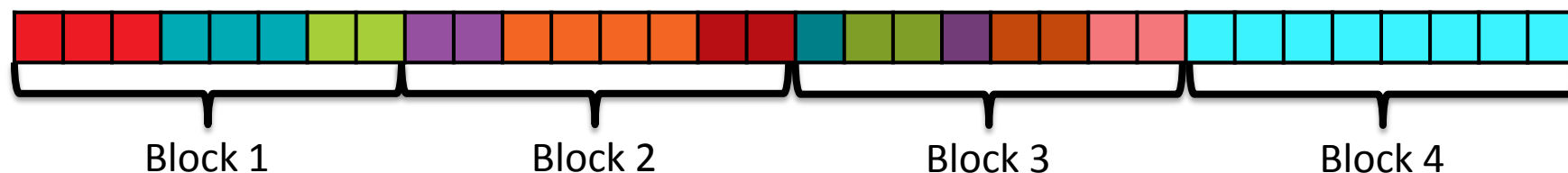
Local Data Share



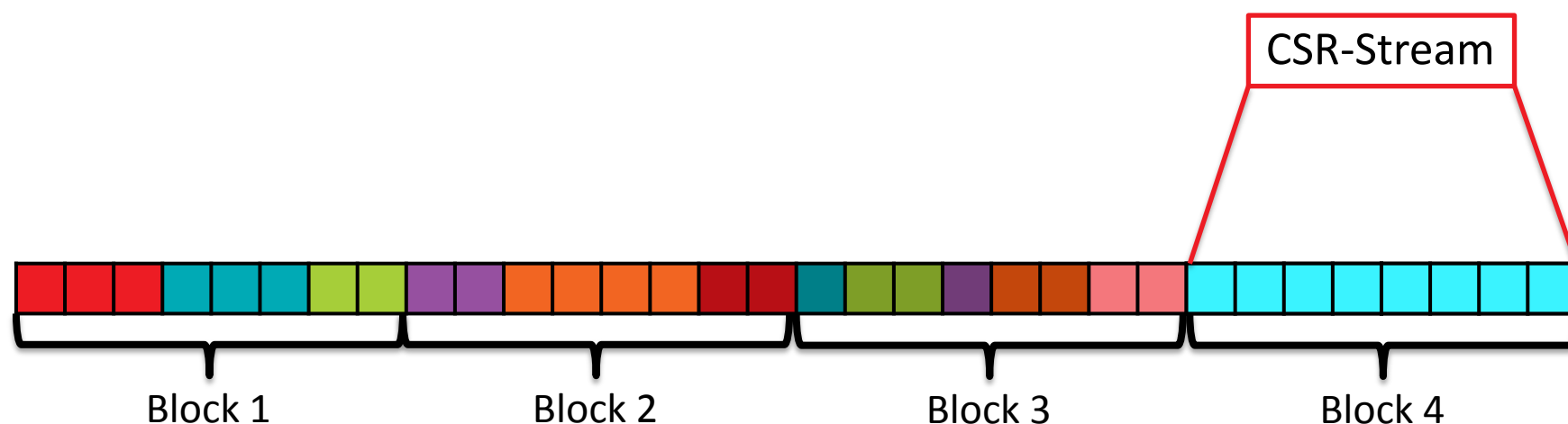
Fast accesses
everywhere



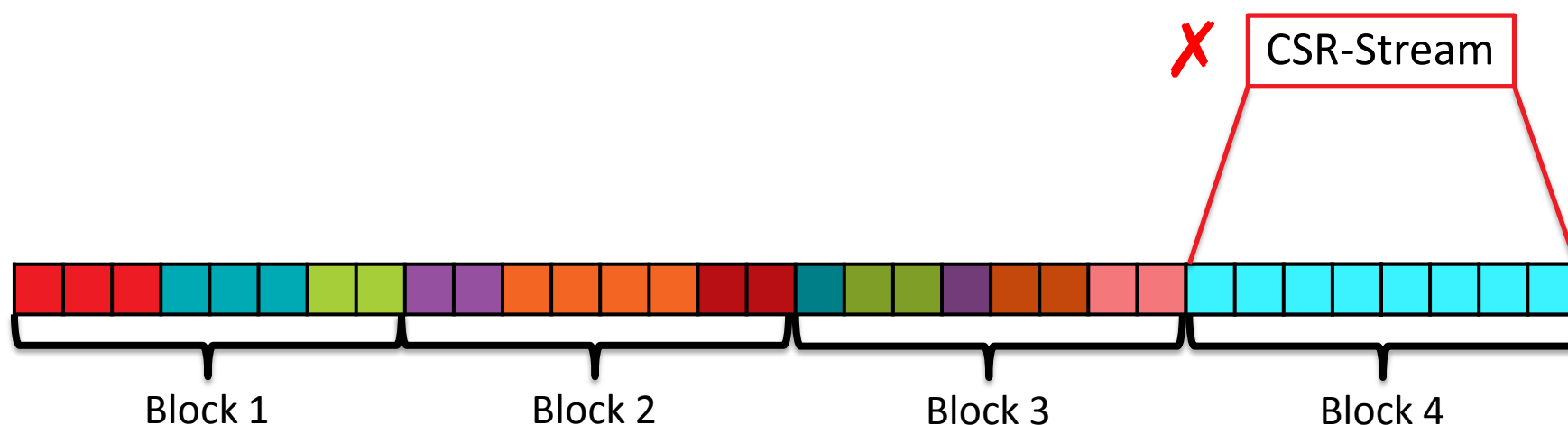
▲ How can we get good bandwidth for “medium-sized” rows?



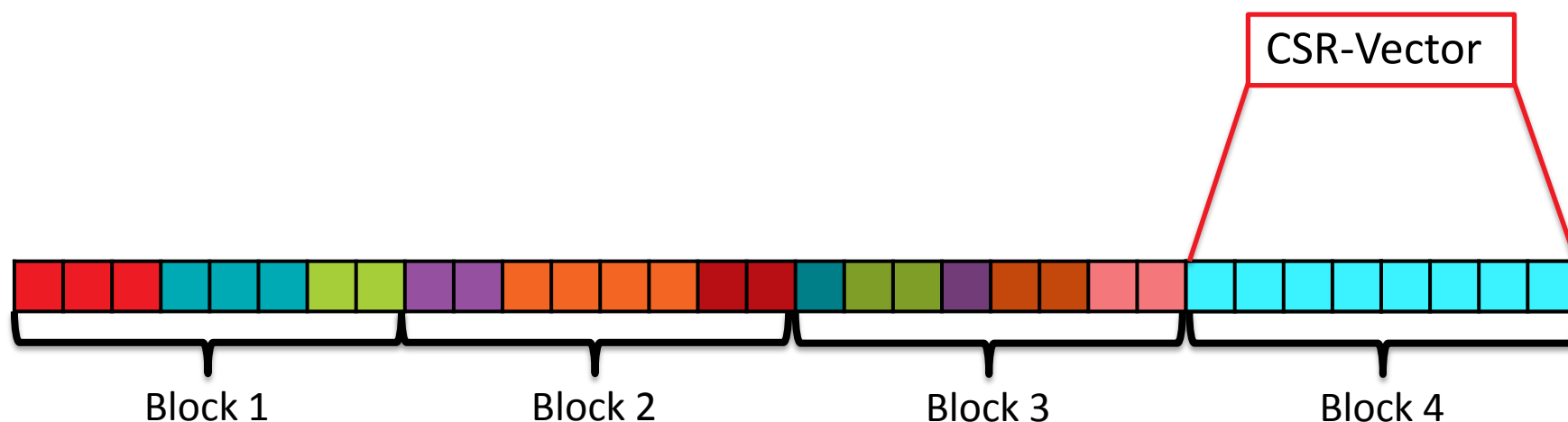
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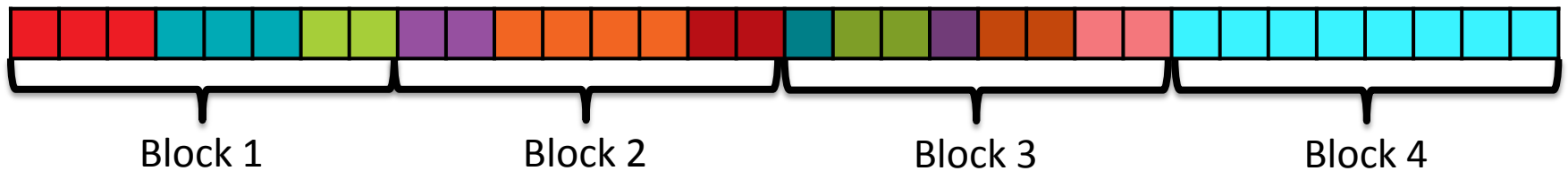
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Efficient use of memory subsystem



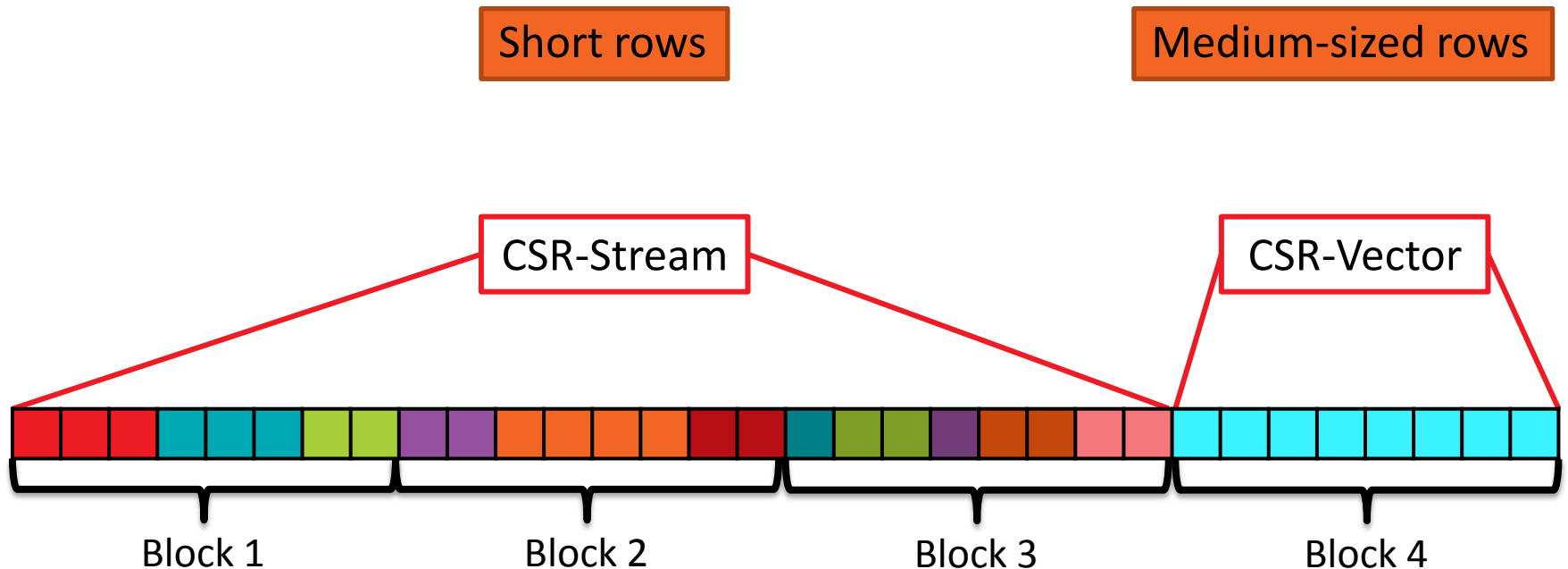
CSR-ADAPTIVE: SHORT AND MEDIUM ROWS



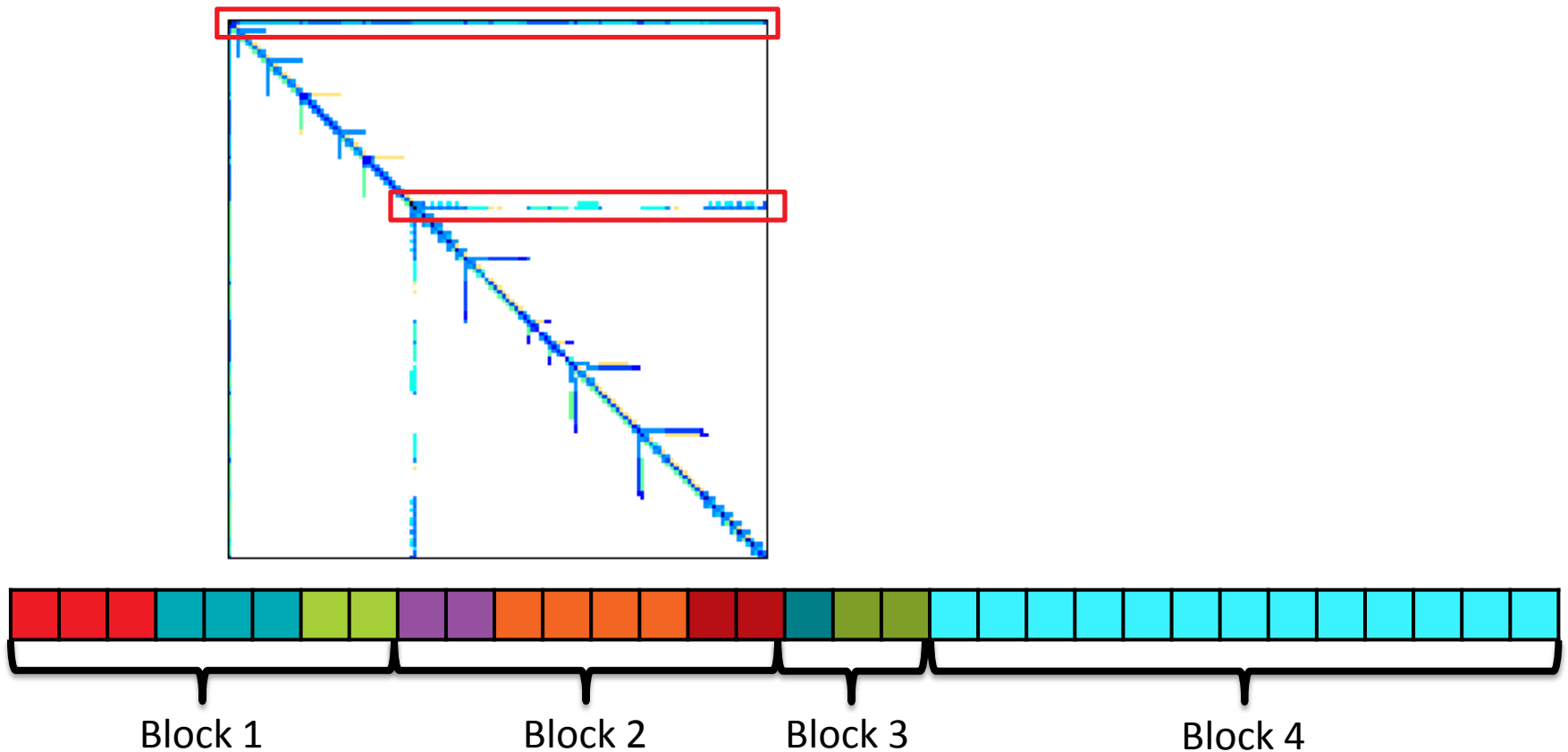
AMD



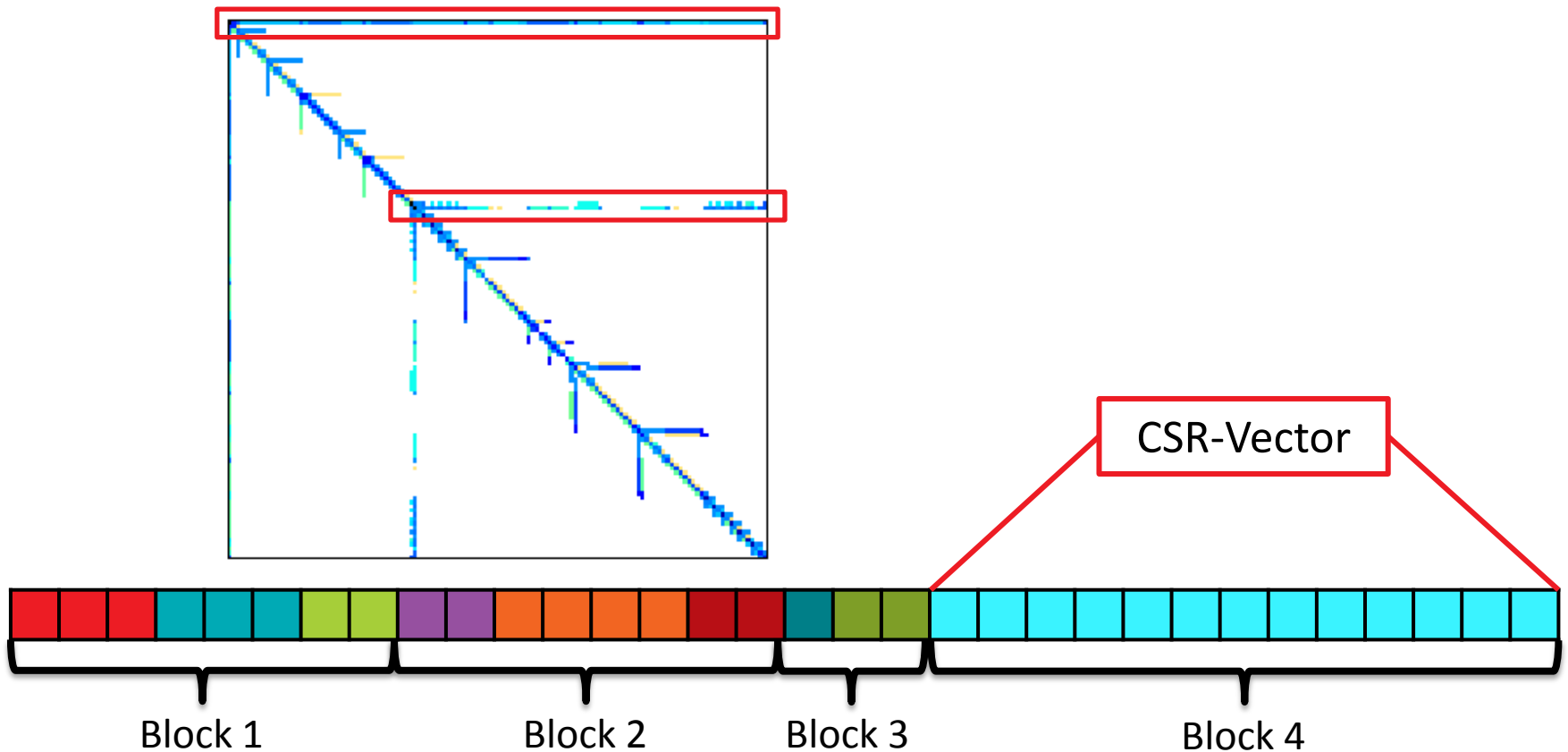
CSR-ADAPTIVE: SHORT AND MEDIUM ROWS



WHAT ABOUT VERY LONG ROWS?



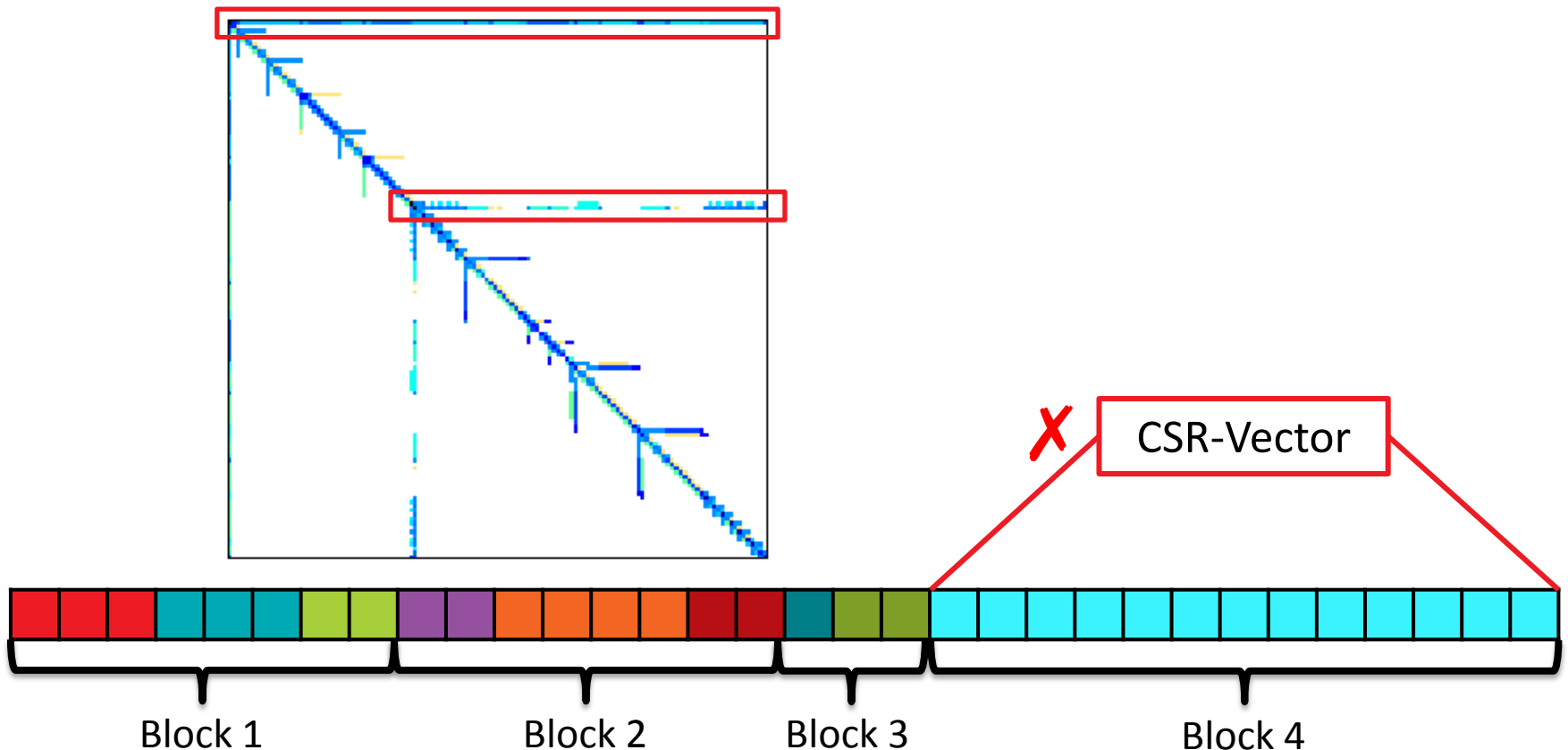
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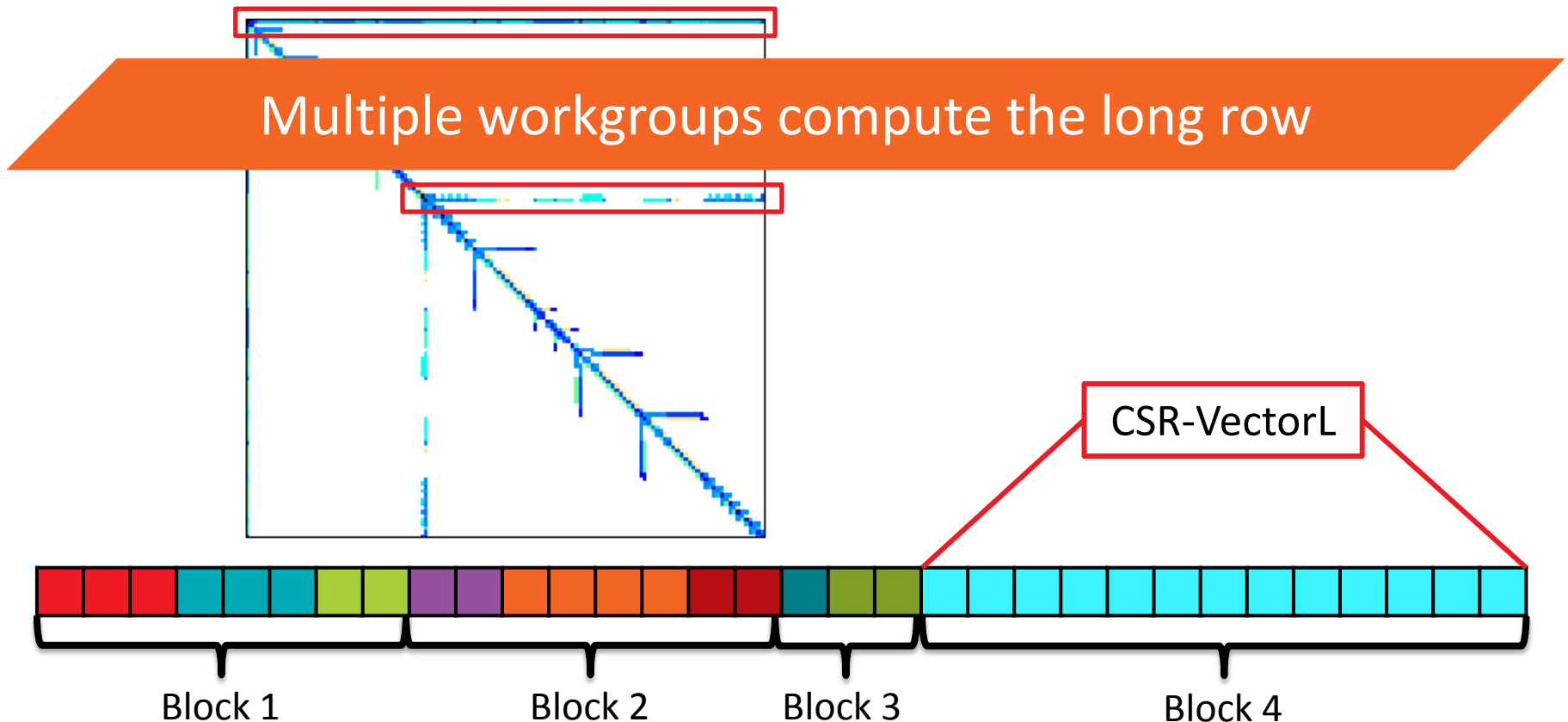
- CSR-Vector: One workgroup can become a performance bottle-neck



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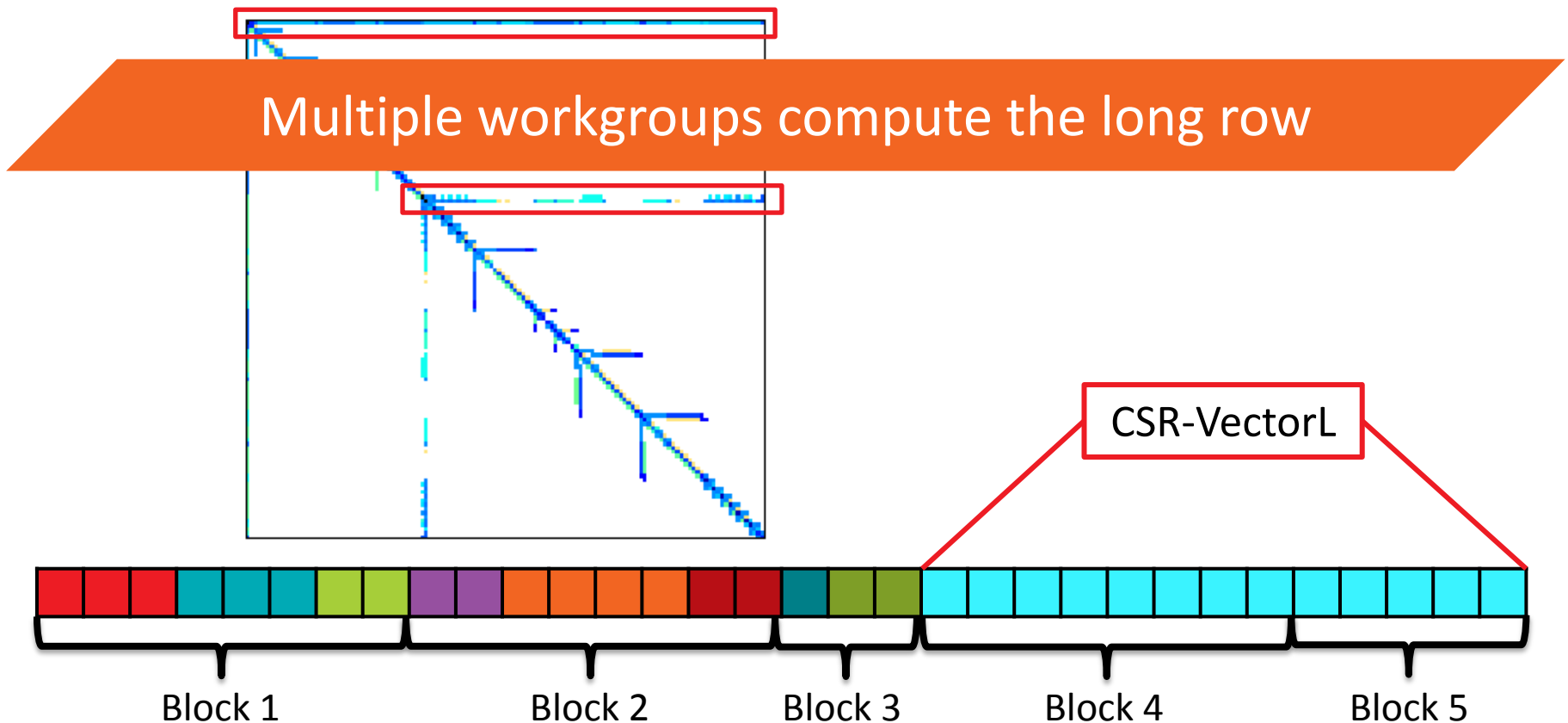
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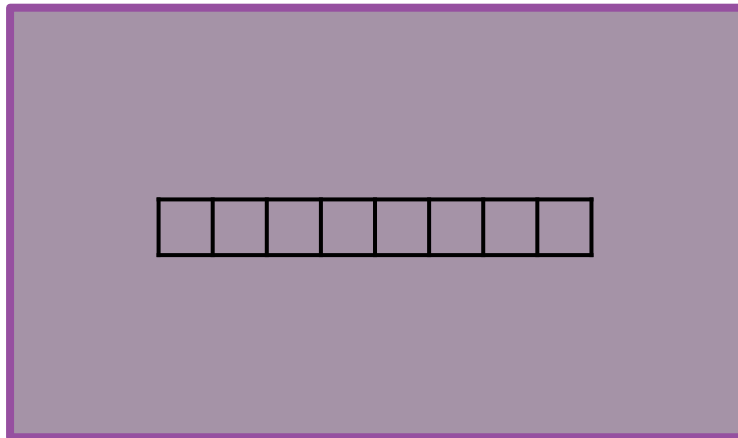
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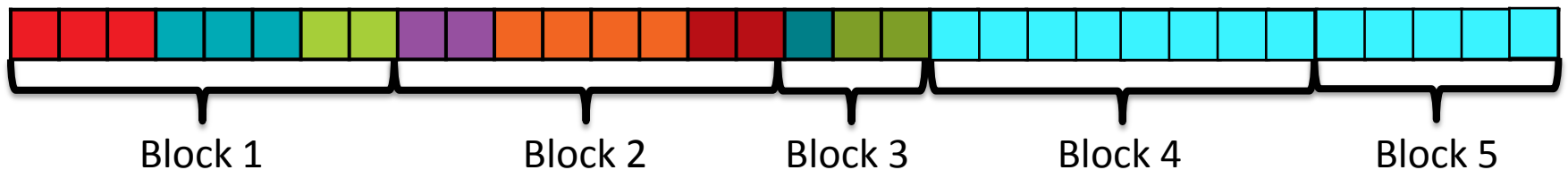
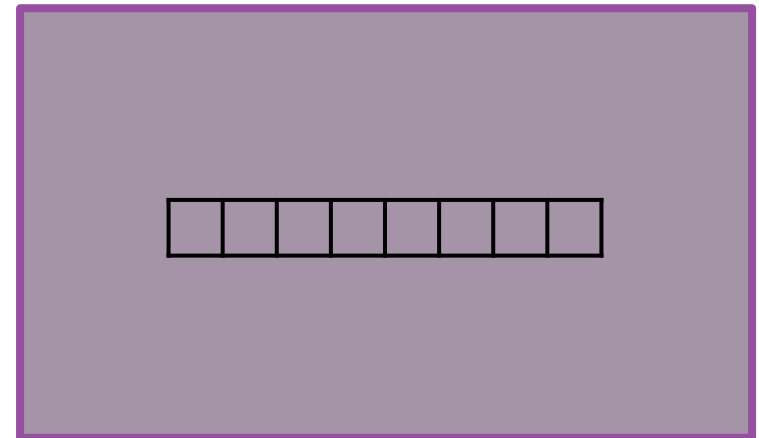
CSR-VECTORL (LONG ROWS)



Local Data Share



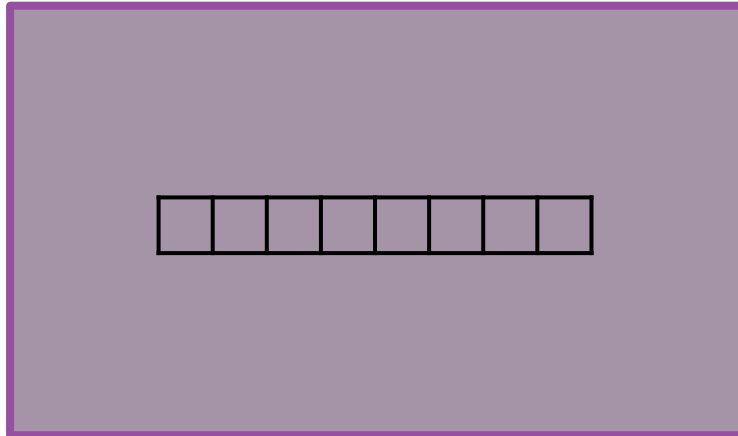
Local Data Share



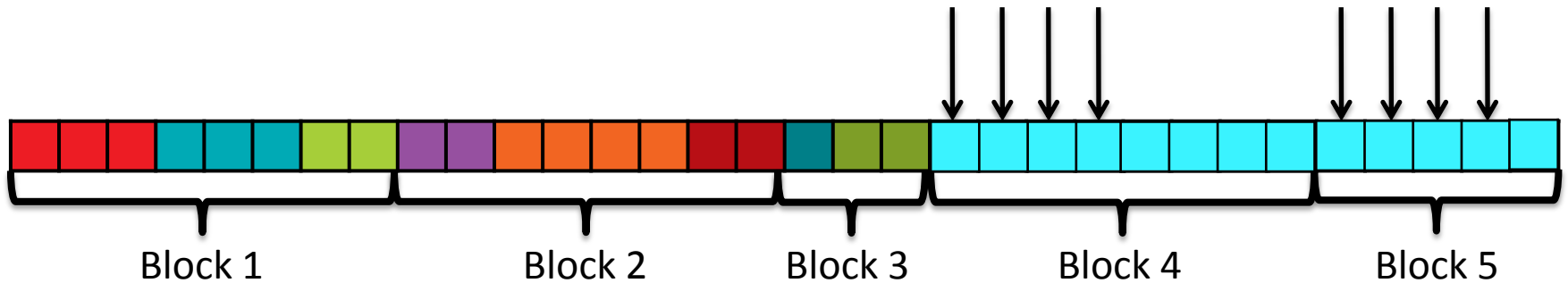
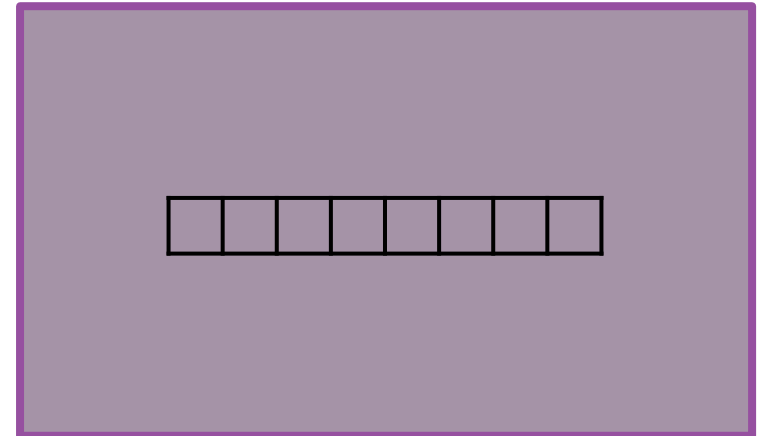
CSR-VECTORL (LONG ROWS)



Local Data Share



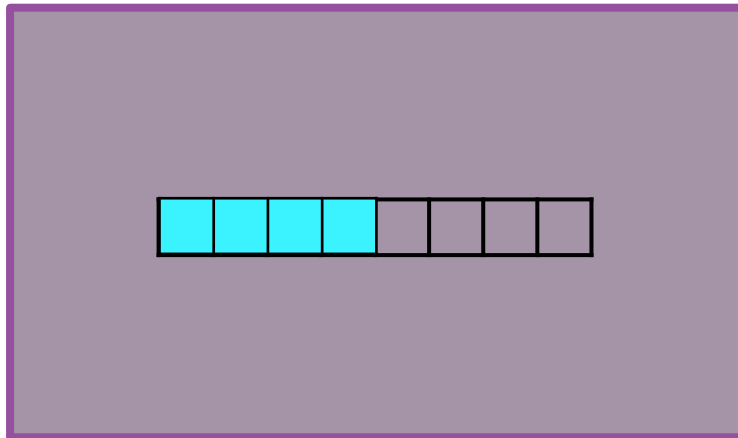
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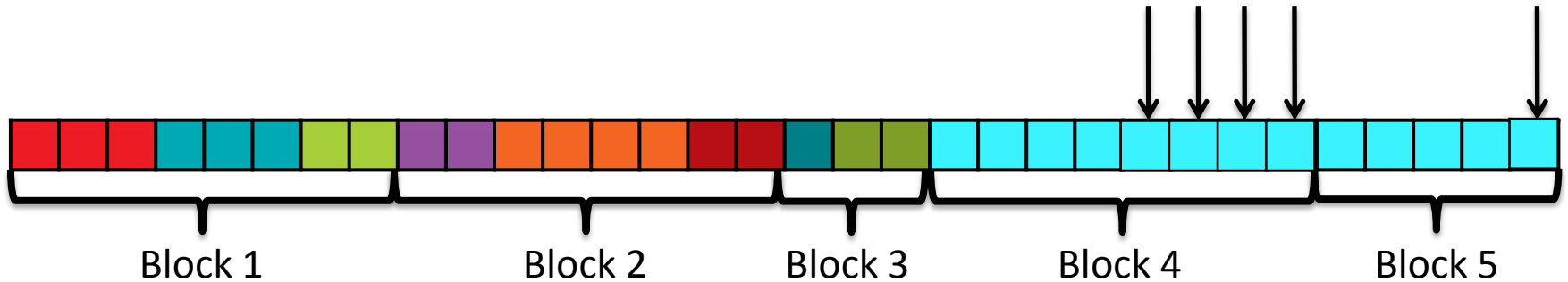
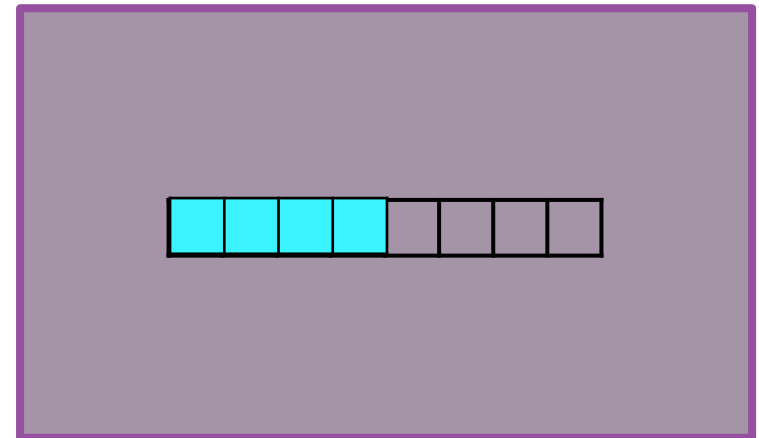
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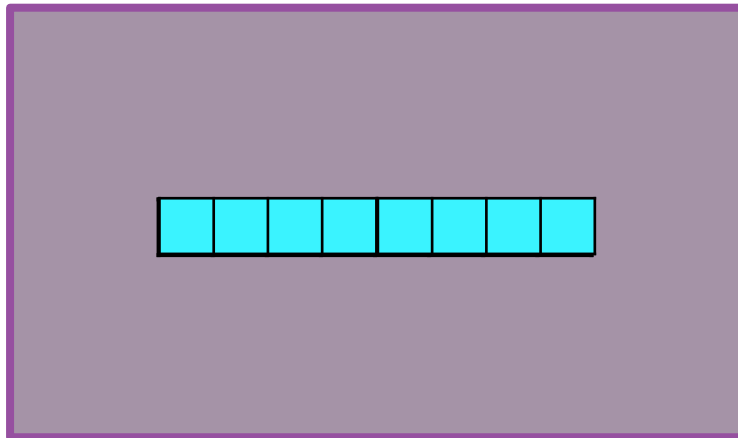
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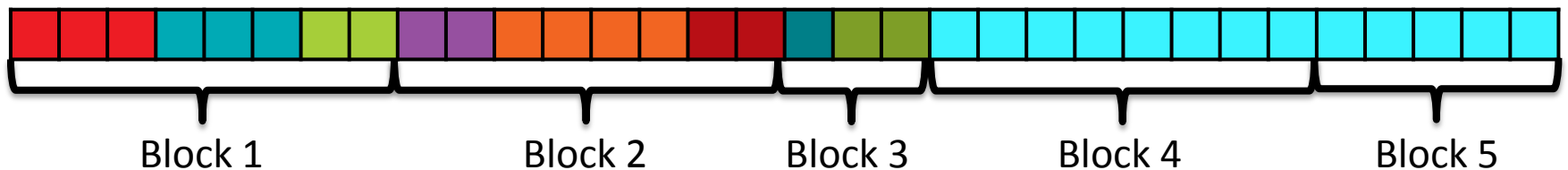
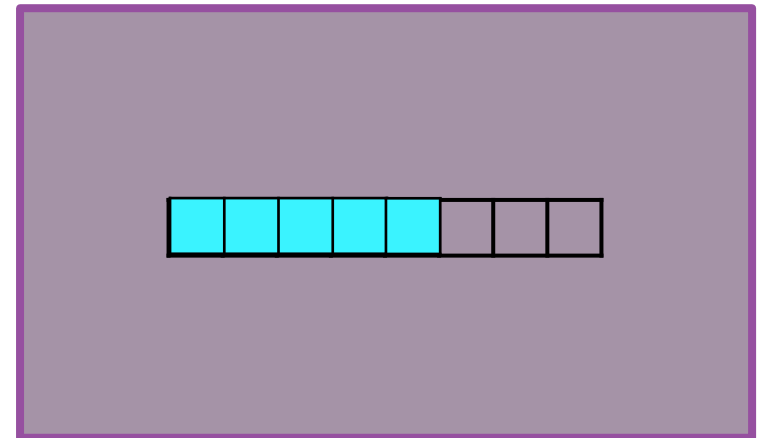
CSR-VECTORL (LONG ROWS)



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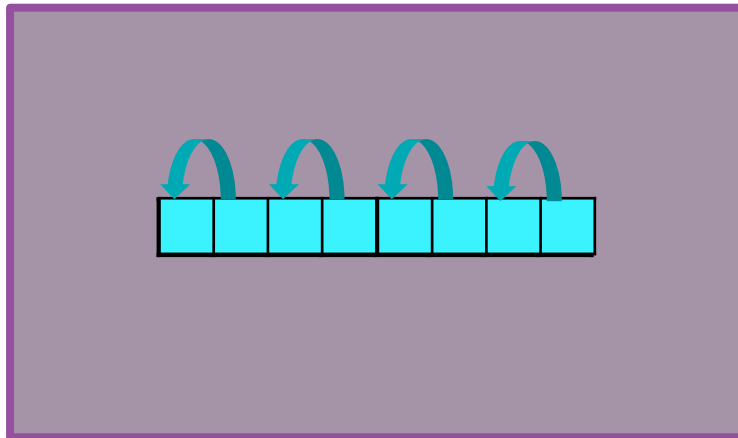
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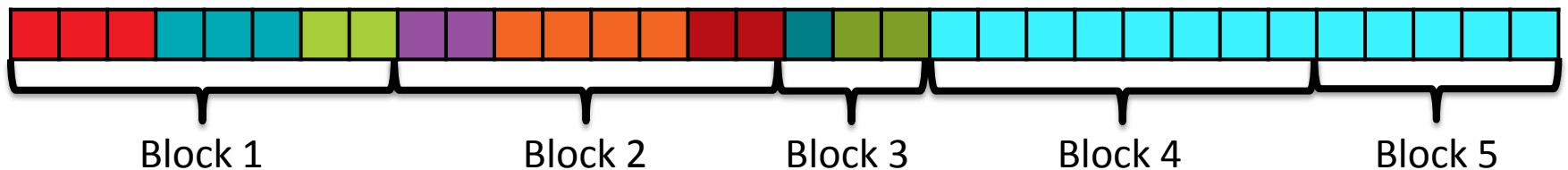
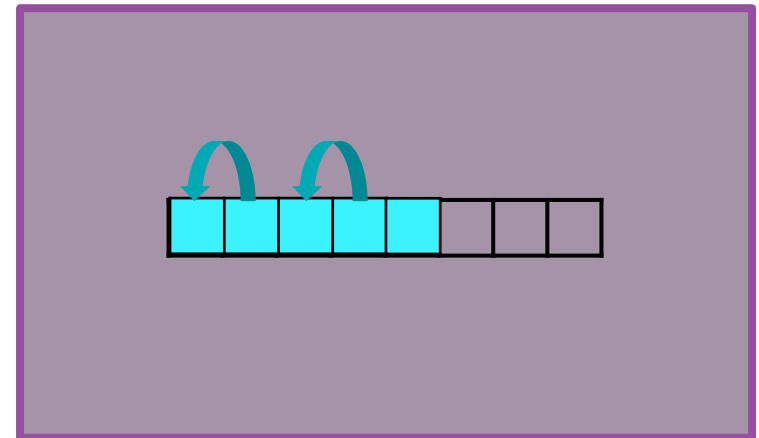
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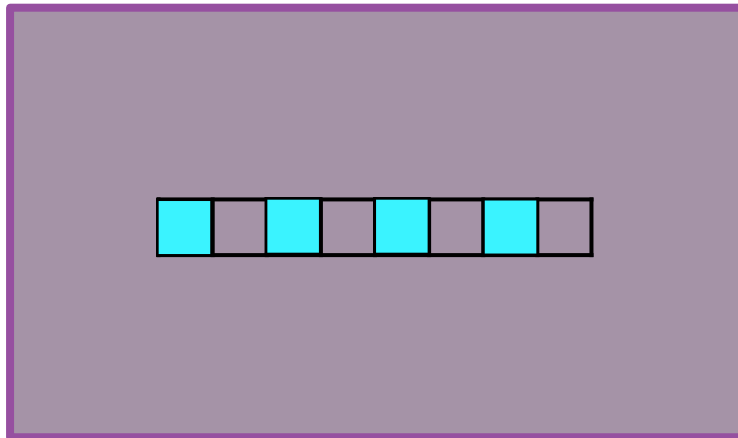
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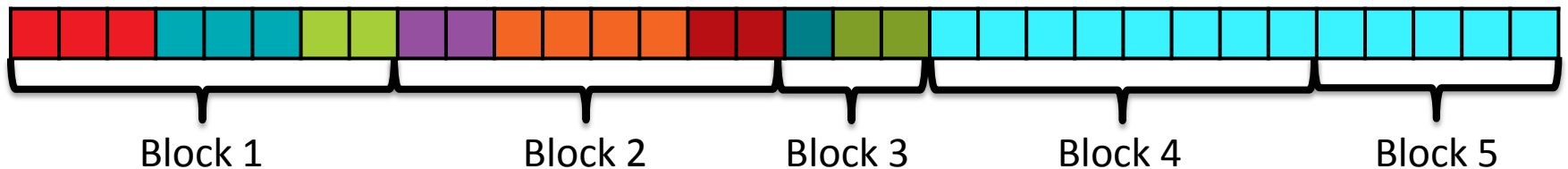
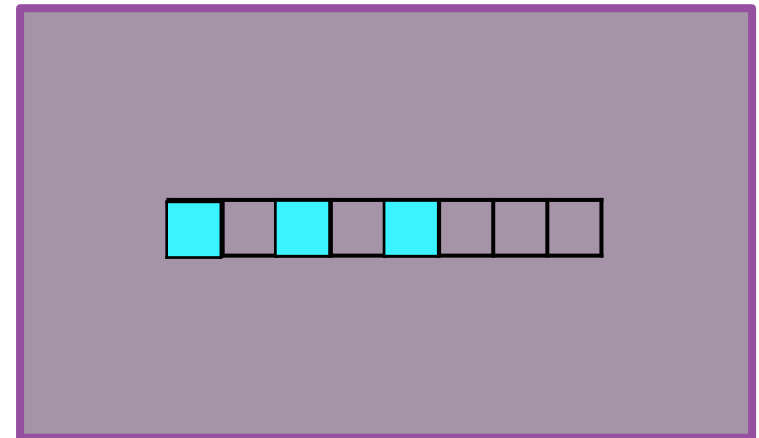
CSR-VECTORL (LONG ROWS)



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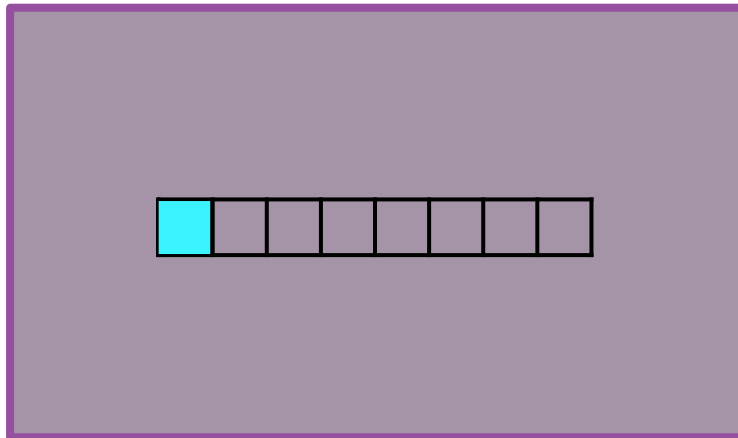
Local Data Share



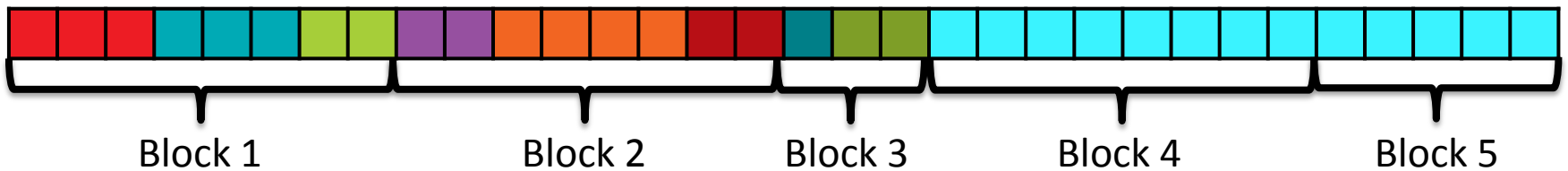
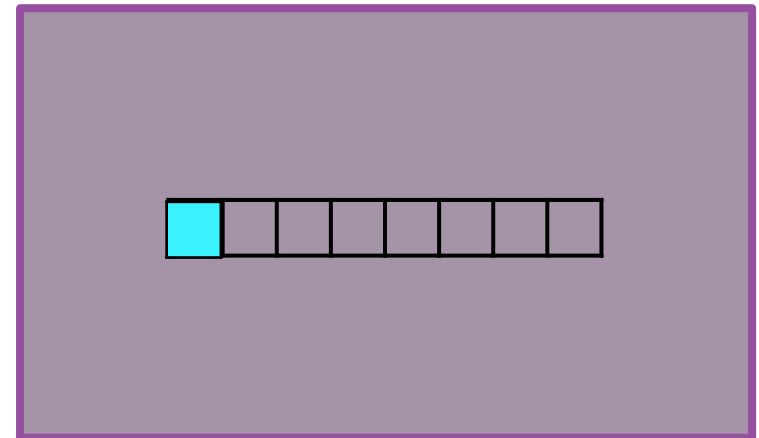
CSR-VECTORL (LONG ROWS)



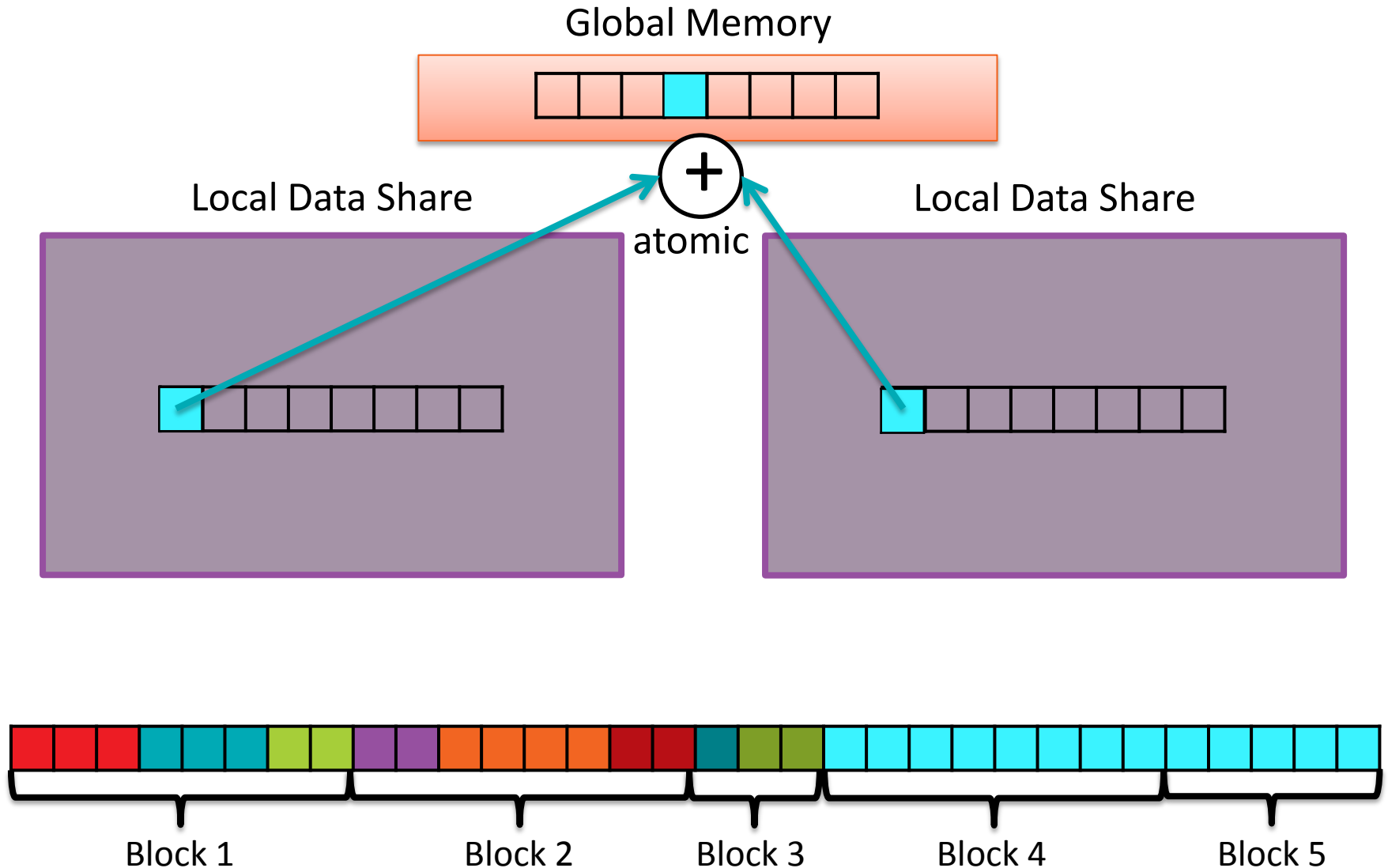
Local Data Share



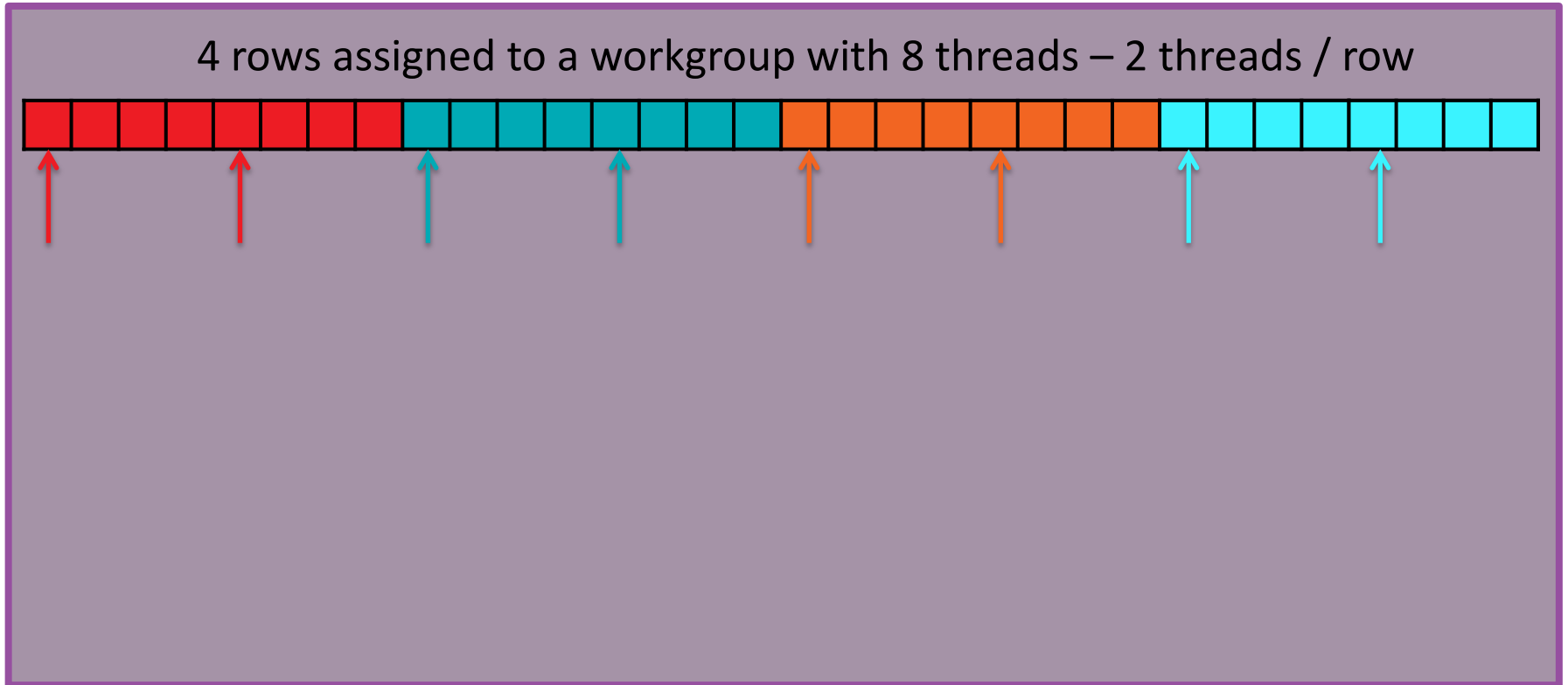
Local Data Share



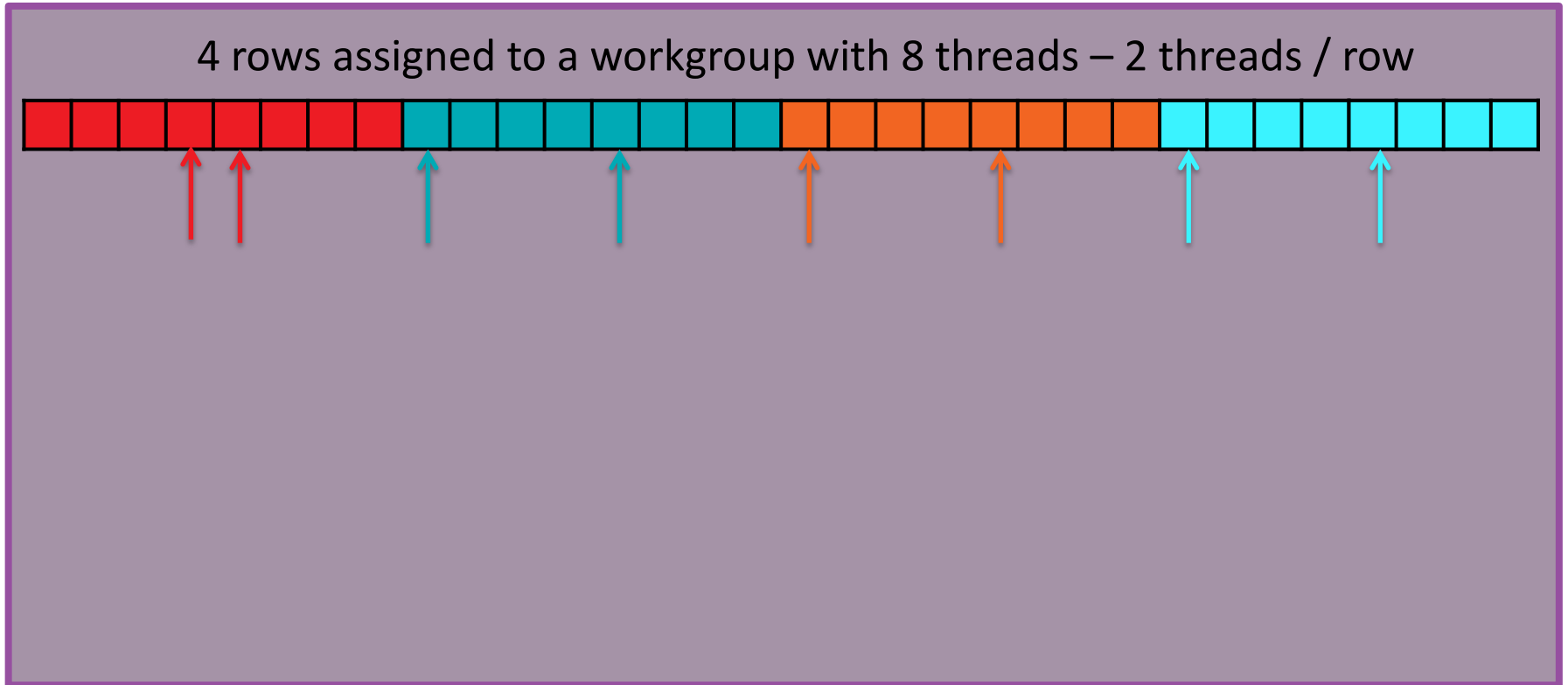
CSR-VECTORL (LONG ROWS)



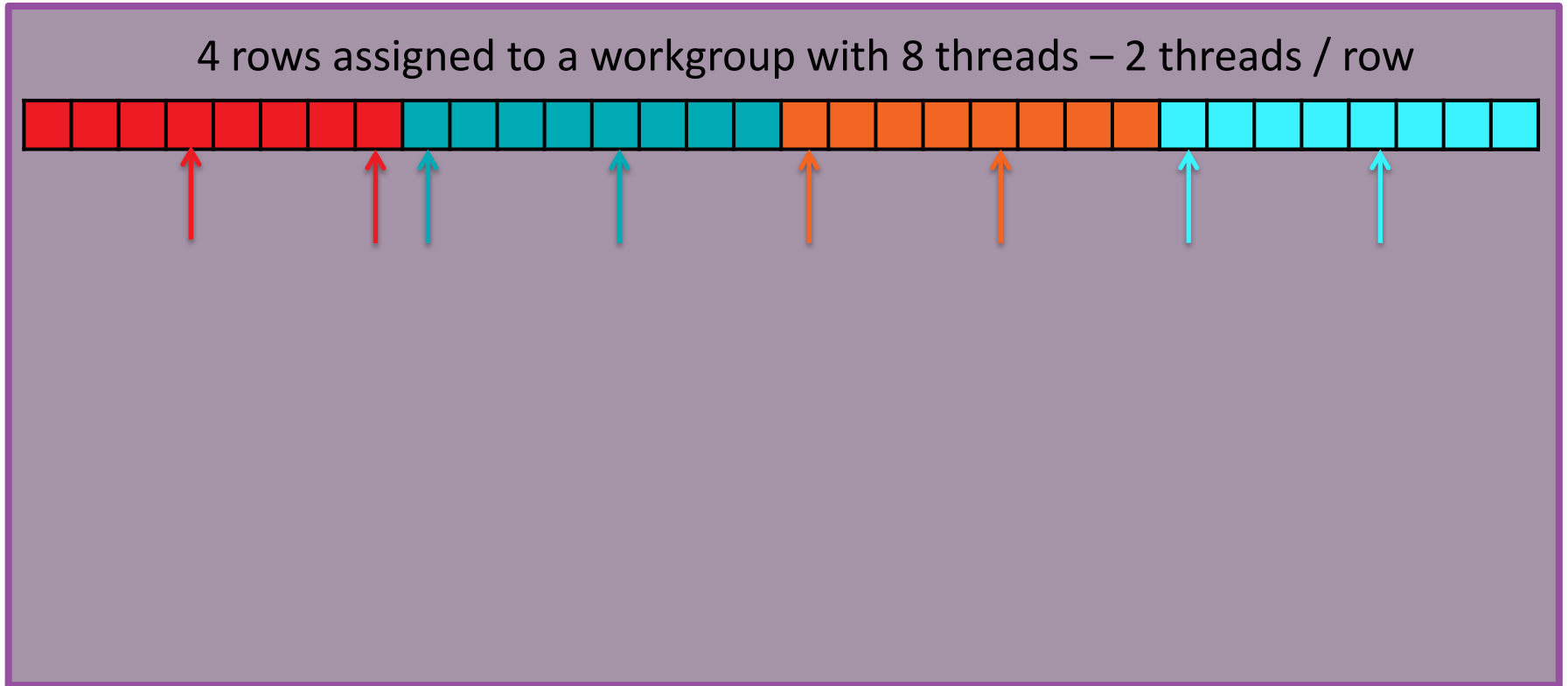
OPTIMIZED CSR-STREAM: LOGARITHMIC REDUCTION



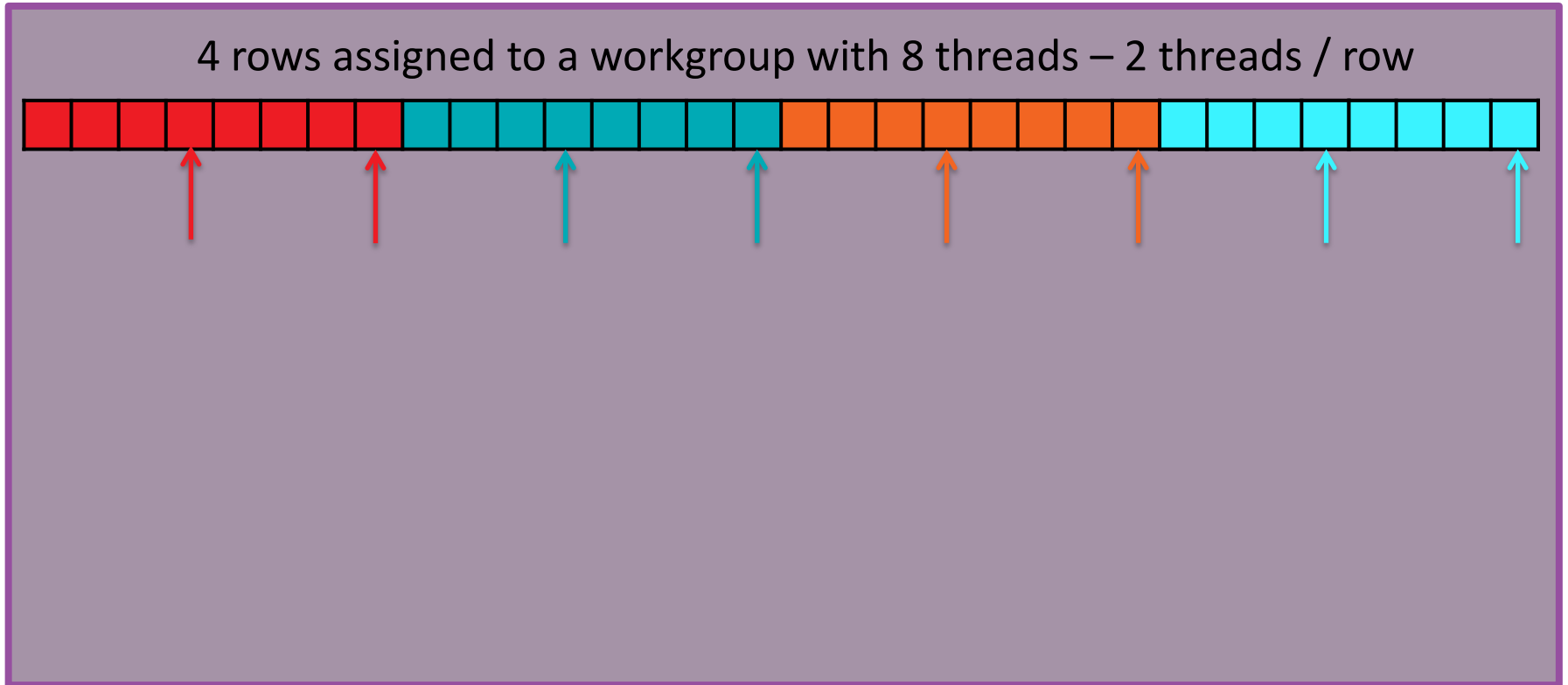
OPTIMIZED CSR-STREAM: LOGARITHMIC REDUCTION



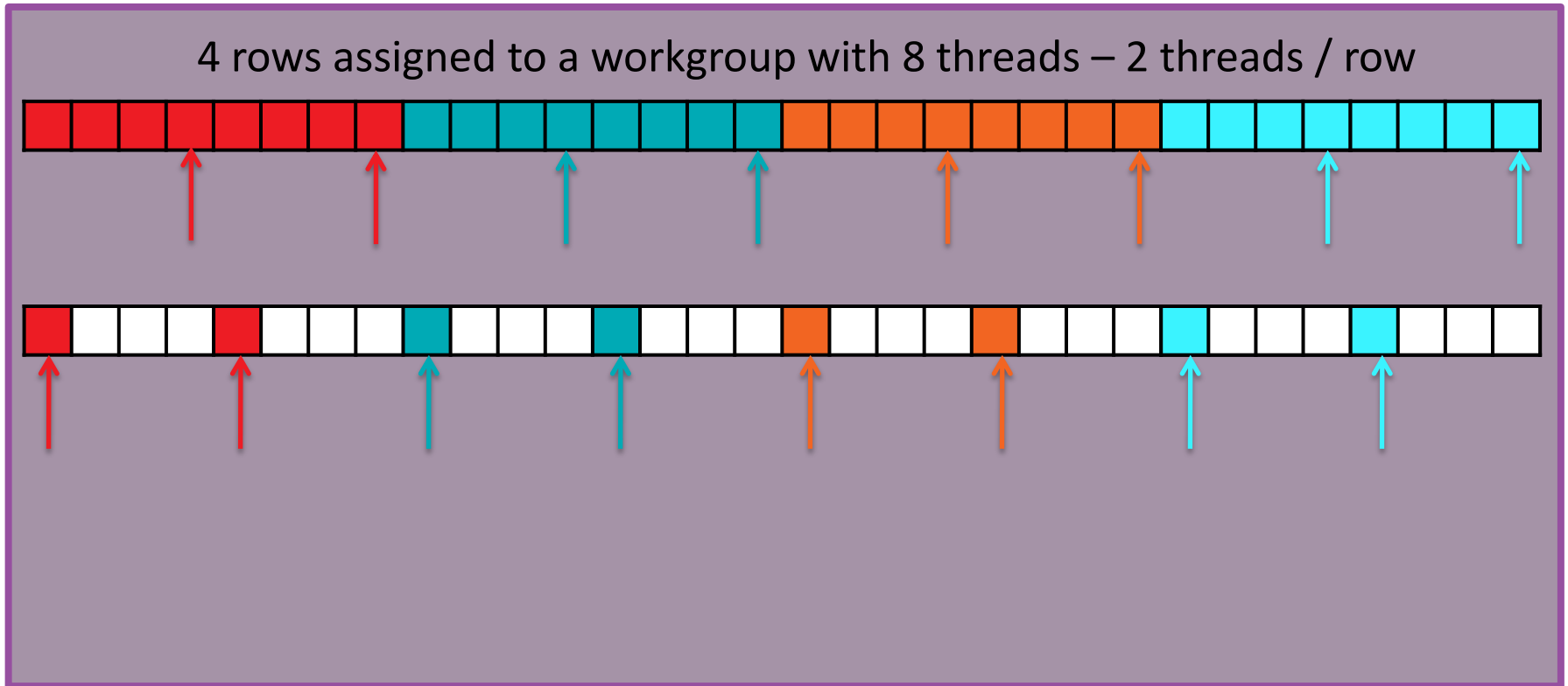
OPTIMIZED CSR-STREAM: LOGARITHMIC REDUCTION



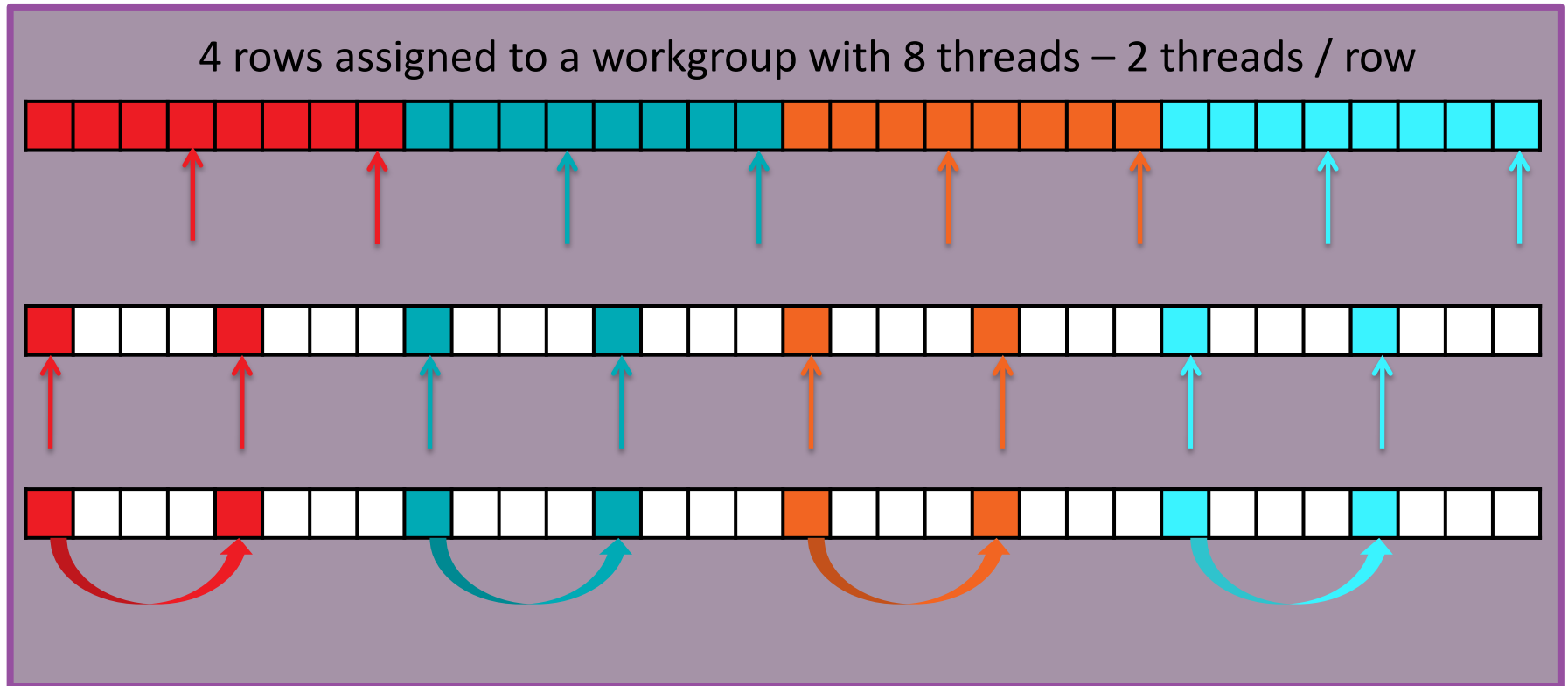
OPTIMIZED CSR-STREAM: LOGARITHMIC REDUCTION

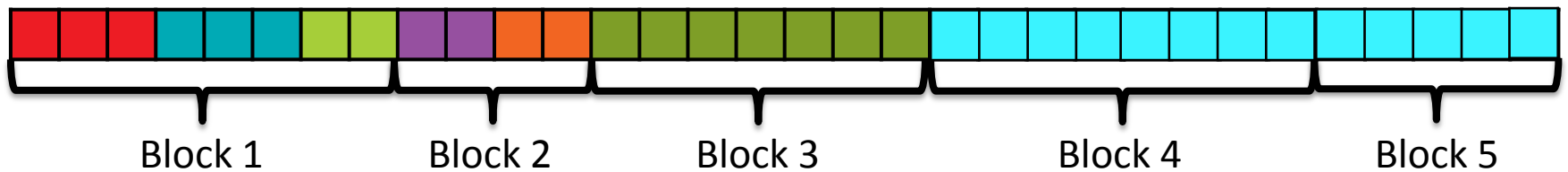


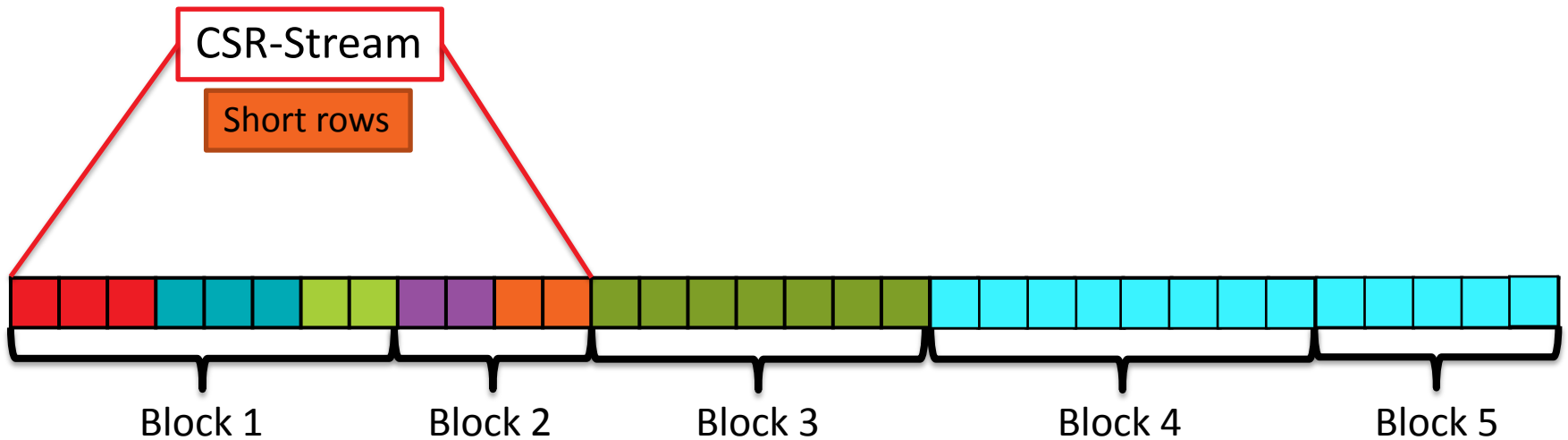
OPTIMIZED CSR-STREAM: LOGARITHMIC REDUCTION

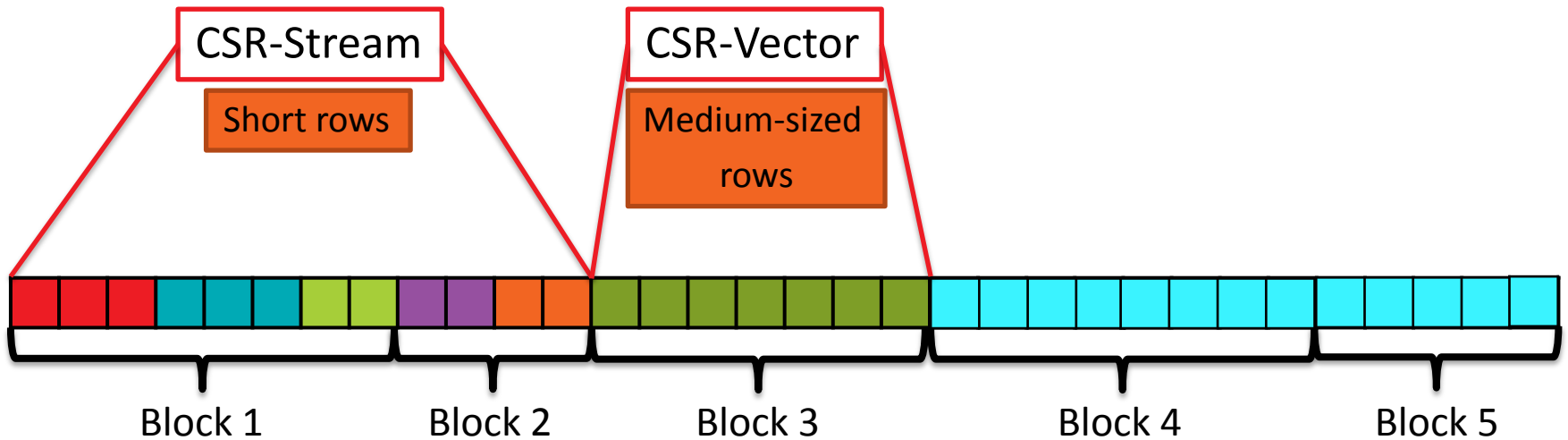


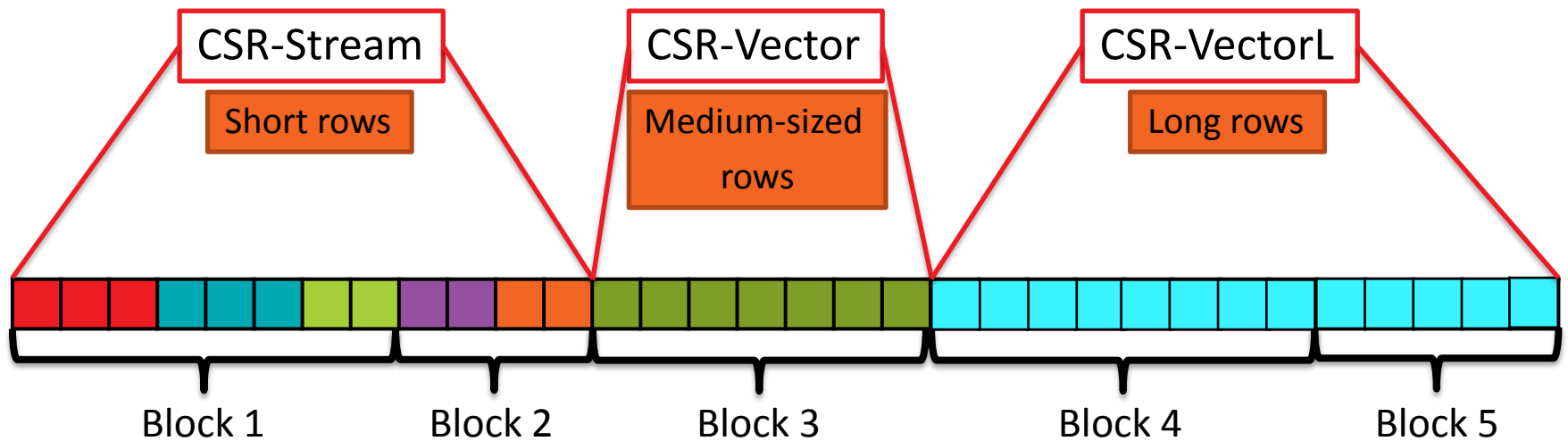
OPTIMIZED CSR-STREAM: LOGARITHMIC REDUCTION



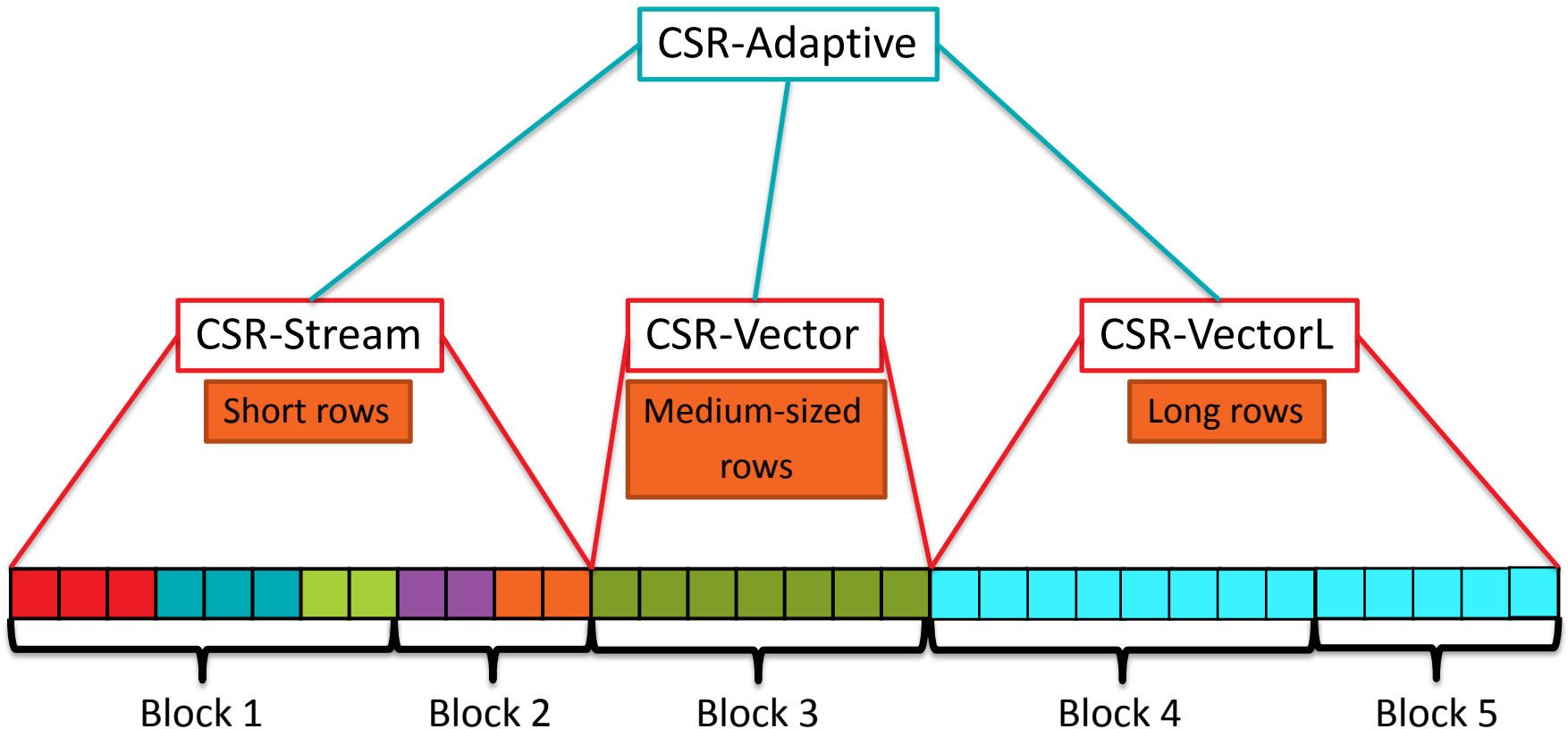








A complete SpMV solution



EXPERIMENTAL SETUP



- ▲ CPU: AMD A10-7850K
 - 32 GB DDR3-2133 SDRAM

- ▲ GPU: AMD FirePro™ W9100
 - 44 parallel 64-wide compute units (CUs) – (2816 ALUs)
 - 5.2 single-precision TFLOPs / 2.6 double-precision TFLOPS
 - 320 GB/s

- ▲ 32 different input matrices used in previous works

- ▲ AMD APP SDK v2.9
- ▲ AMD FirePro driver v14.20 Beta on Ubuntu 14.04.2

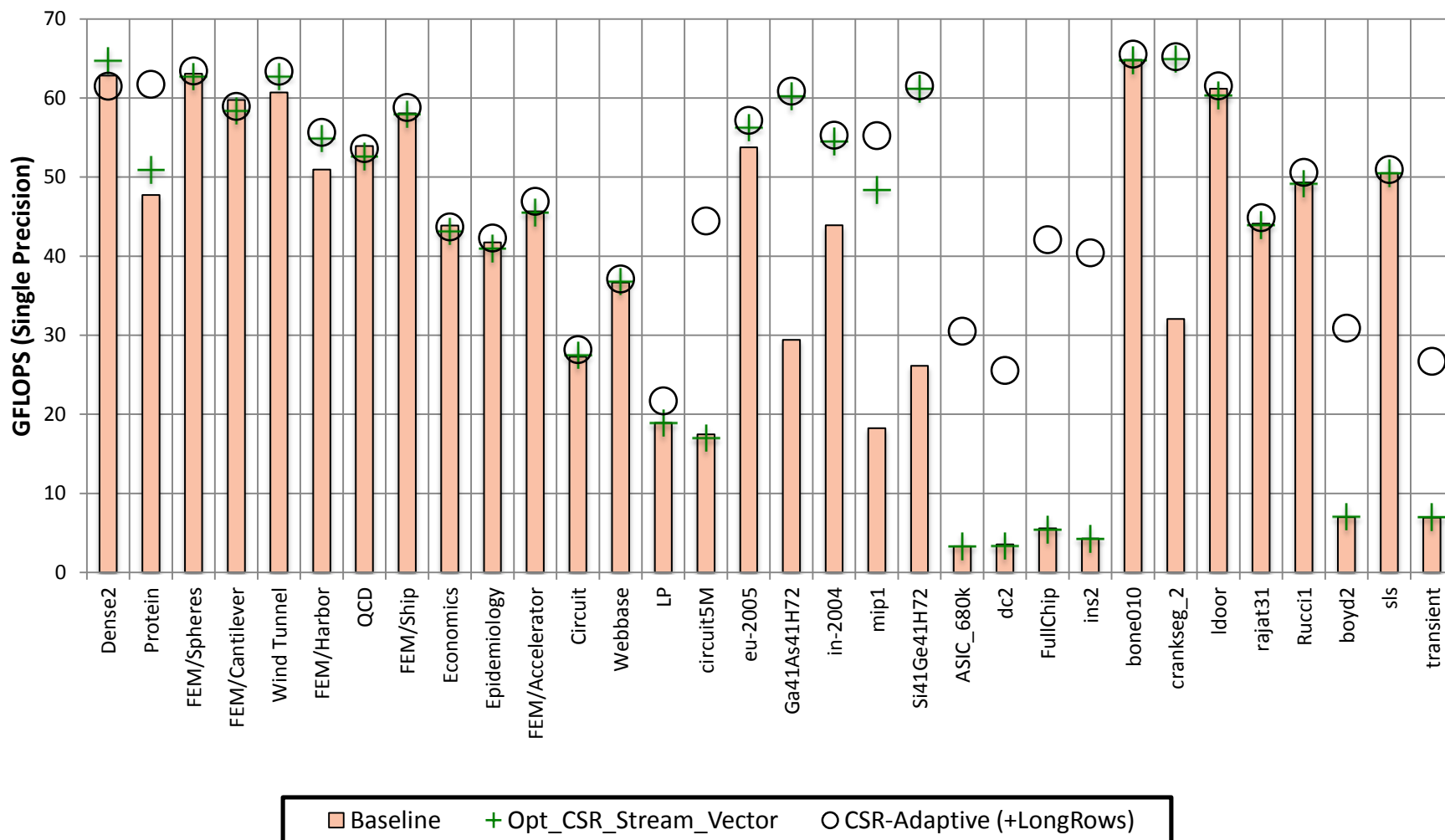
CSR-ADAPTIVE OPTIMIZATIONS

AMD FIREPRO W9100 GPU



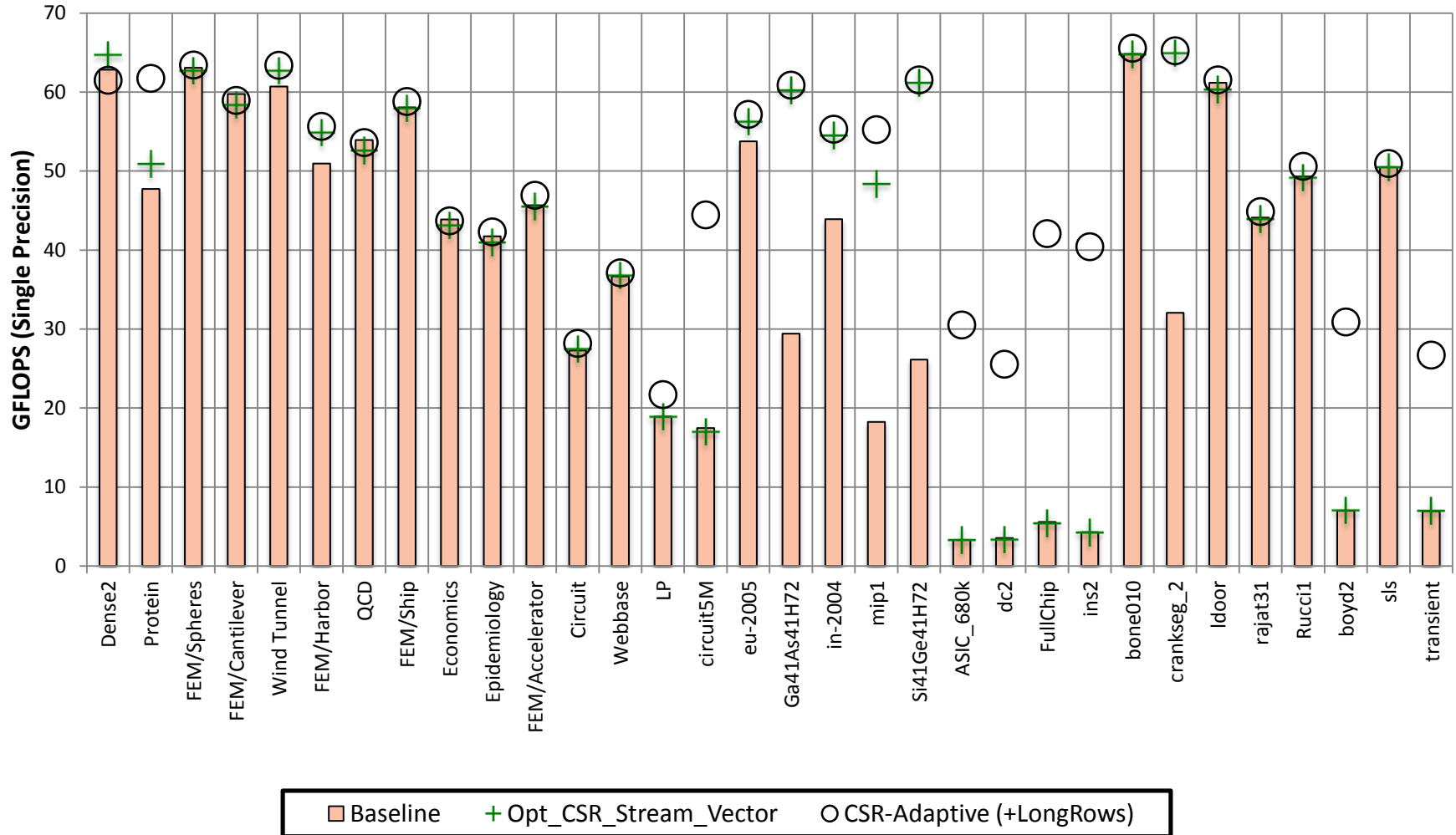
CSR-ADAPTIVE OPTIMIZATIONS

AMD FIREPRO W9100 GPU



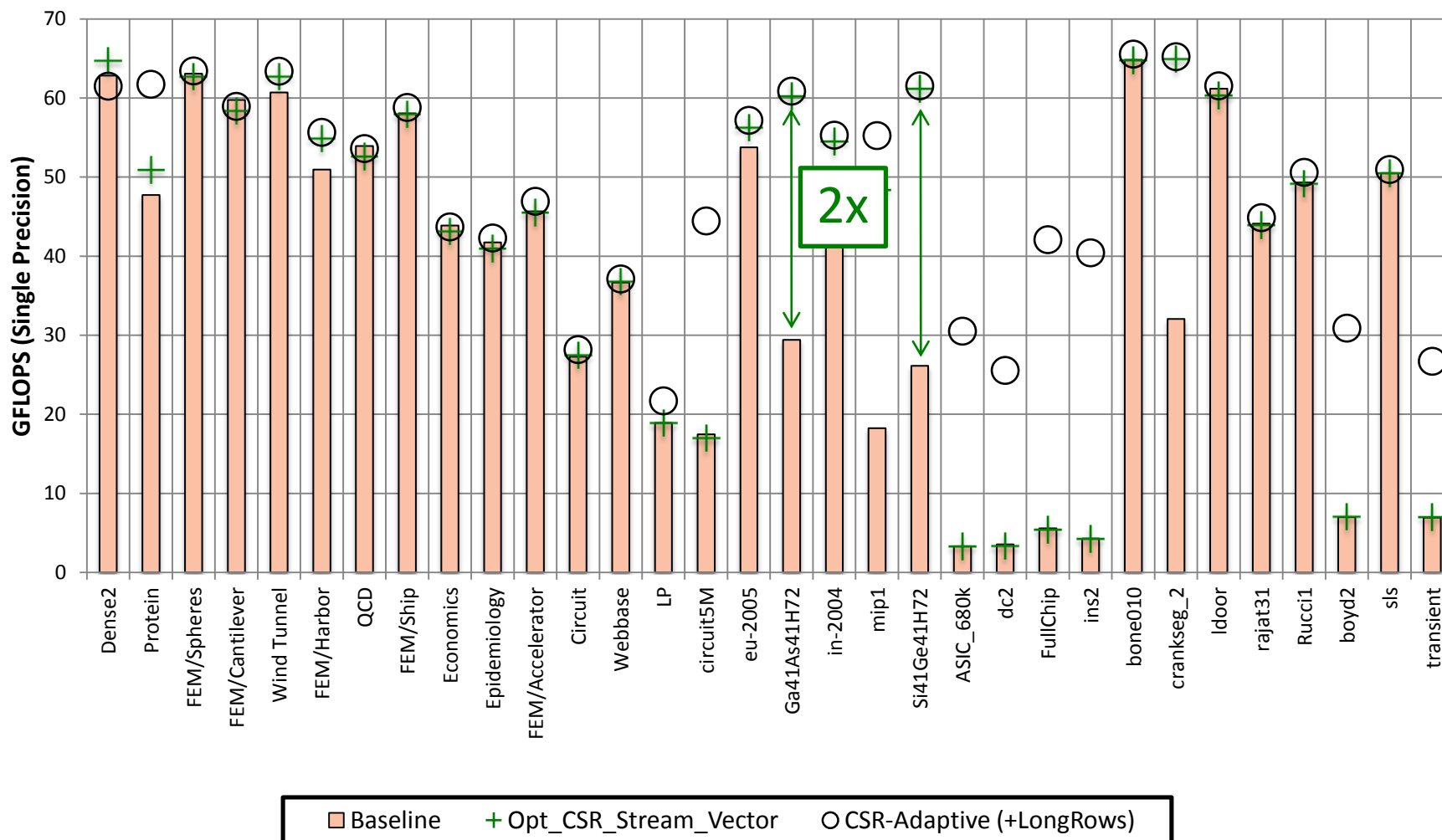
CSR-ADAPTIVE OPTIMIZATIONS

AMD FIREPRO W9100 GPU



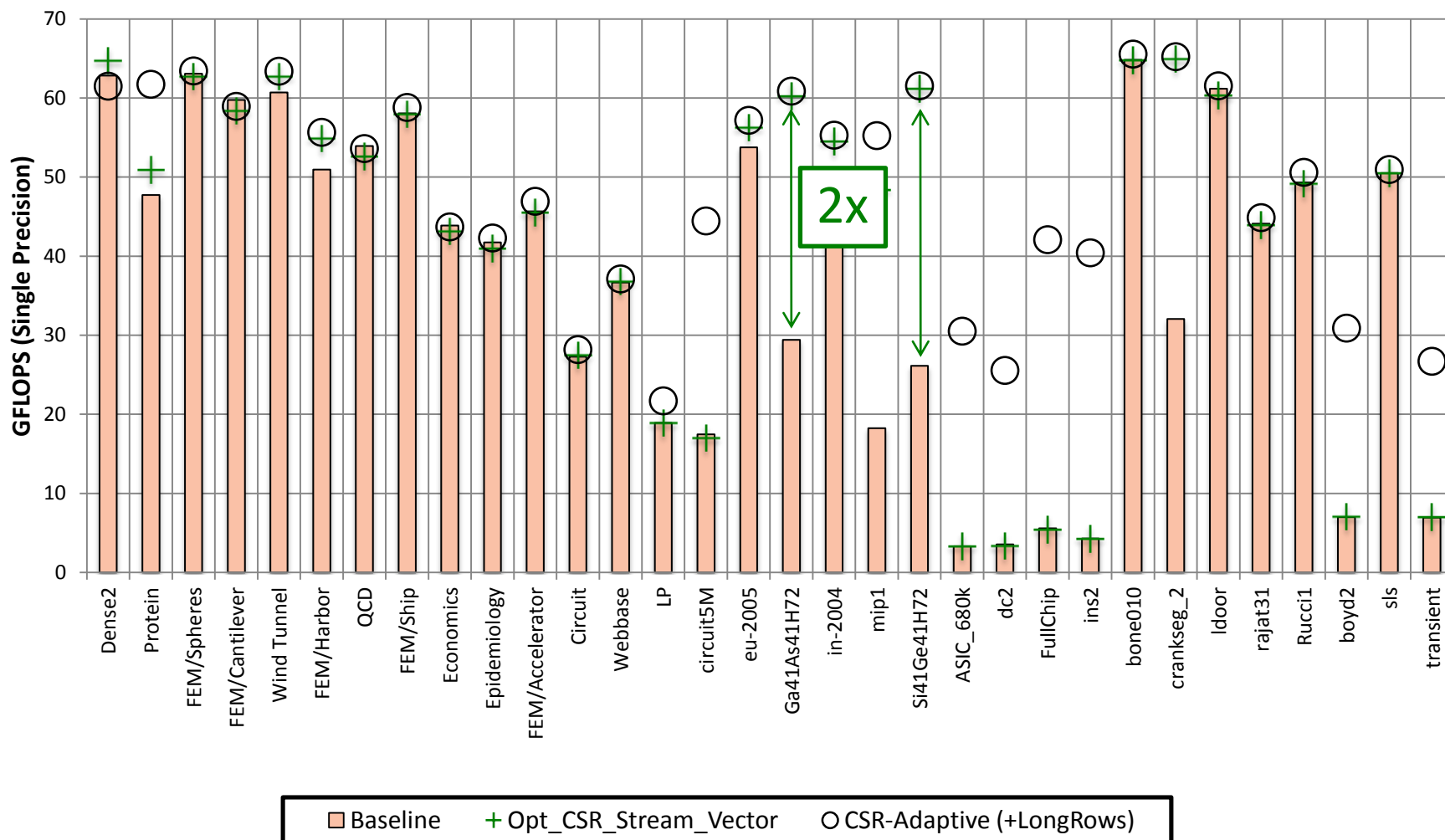
CSR-ADAPTIVE OPTIMIZATIONS

AMD FIREPRO W9100 GPU



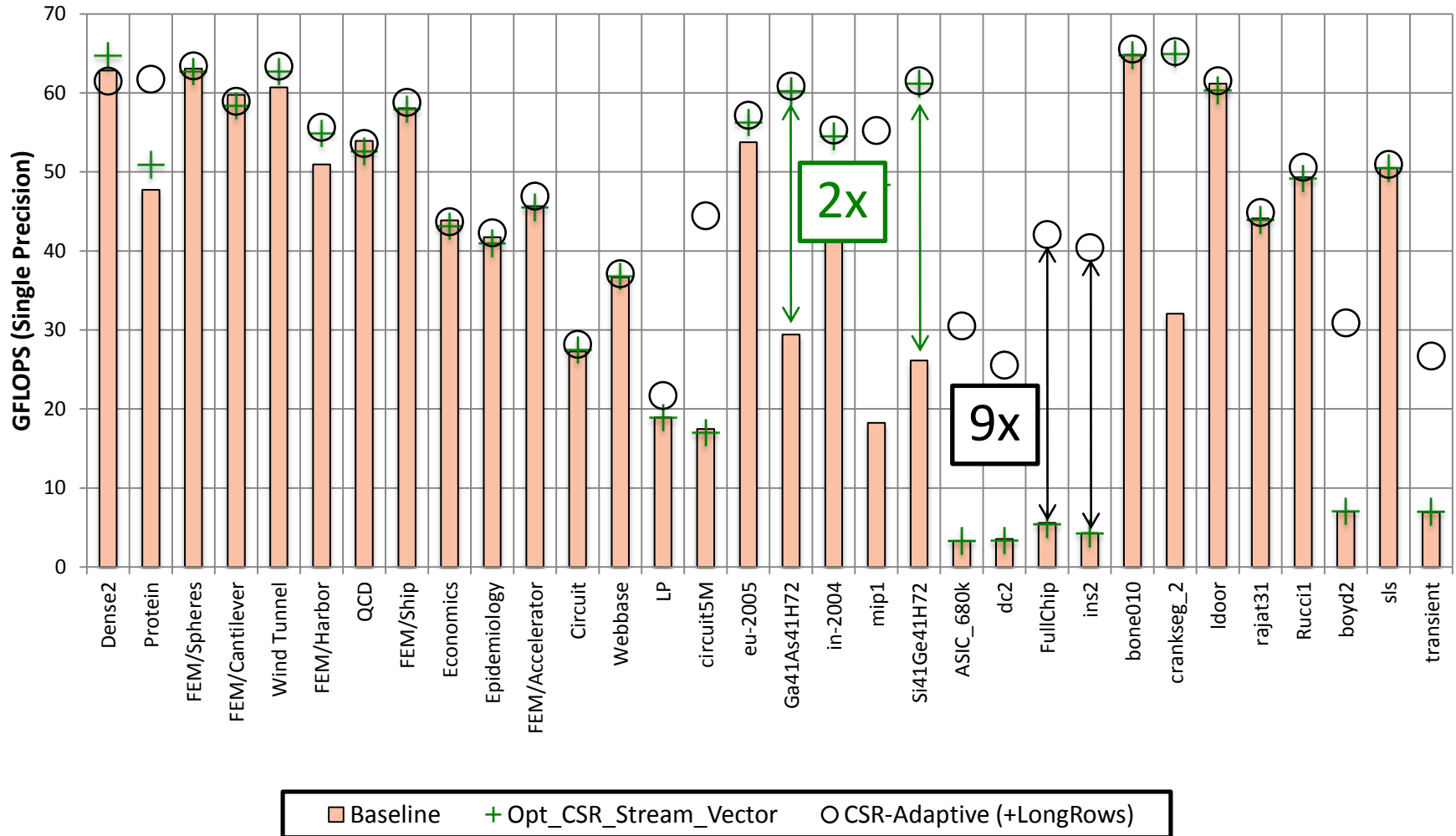
CSR-ADAPTIVE OPTIMIZATIONS

AMD FIREPRO W9100 GPU



CSR-ADAPTIVE OPTIMIZATIONS

AMD FIREPRO W9100 GPU



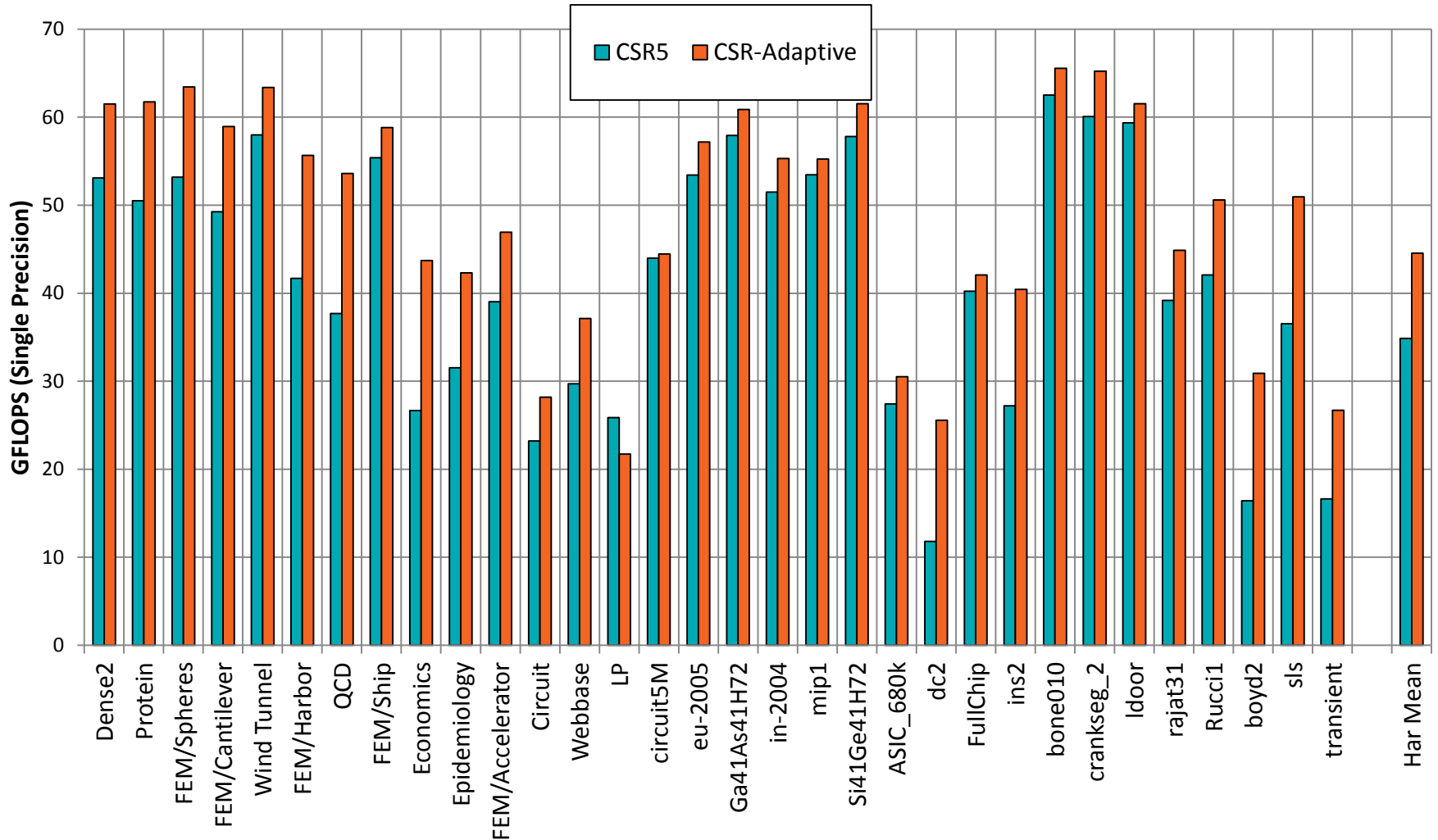
CSR-ADAPTIVE VS CSR5

AMD FIREPRO W9100 GPU (SINGLE PRECISION)



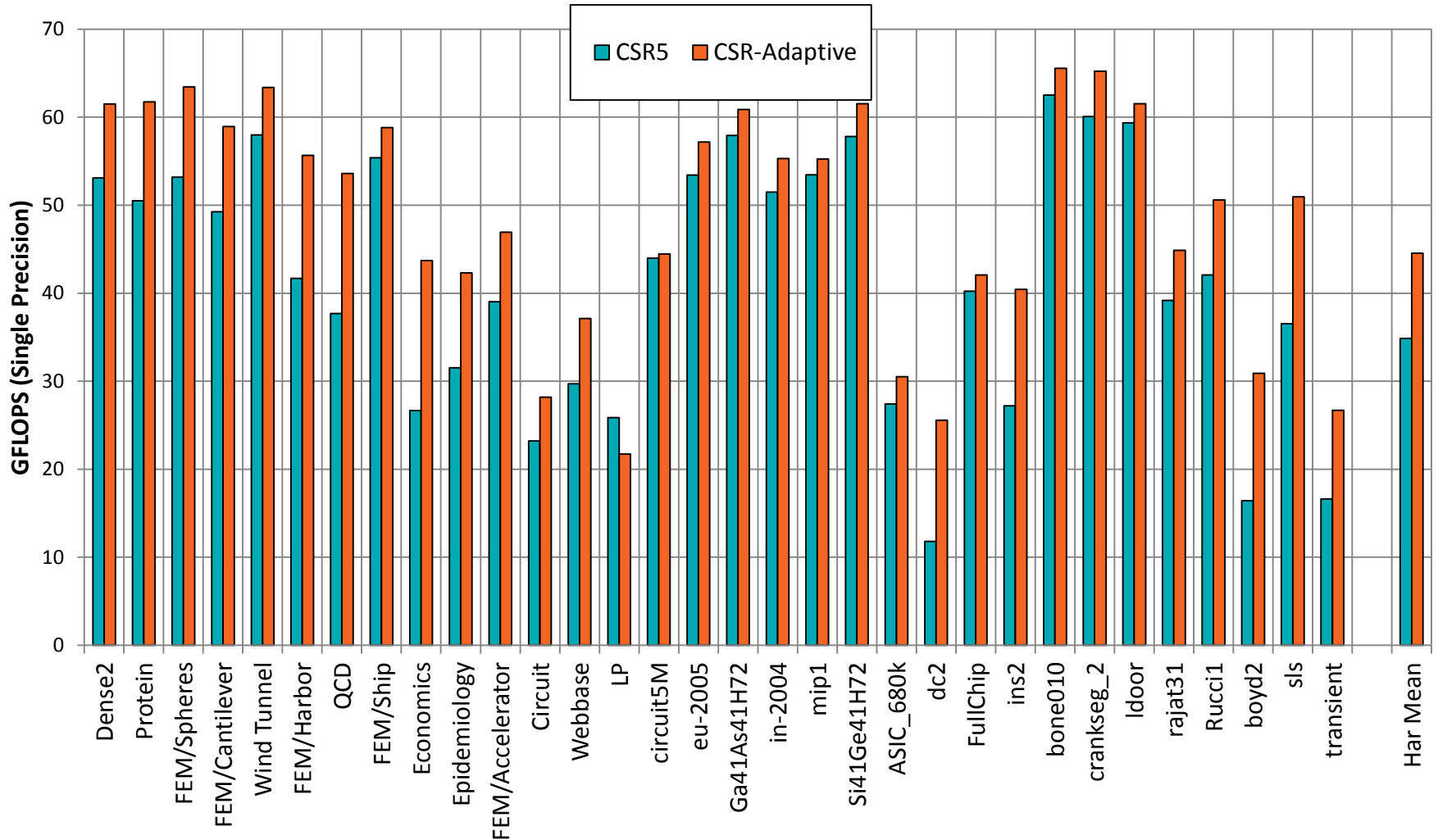
CSR-ADAPTIVE VS CSR5

AMD FIREPRO W9100 GPU (SINGLE PRECISION)



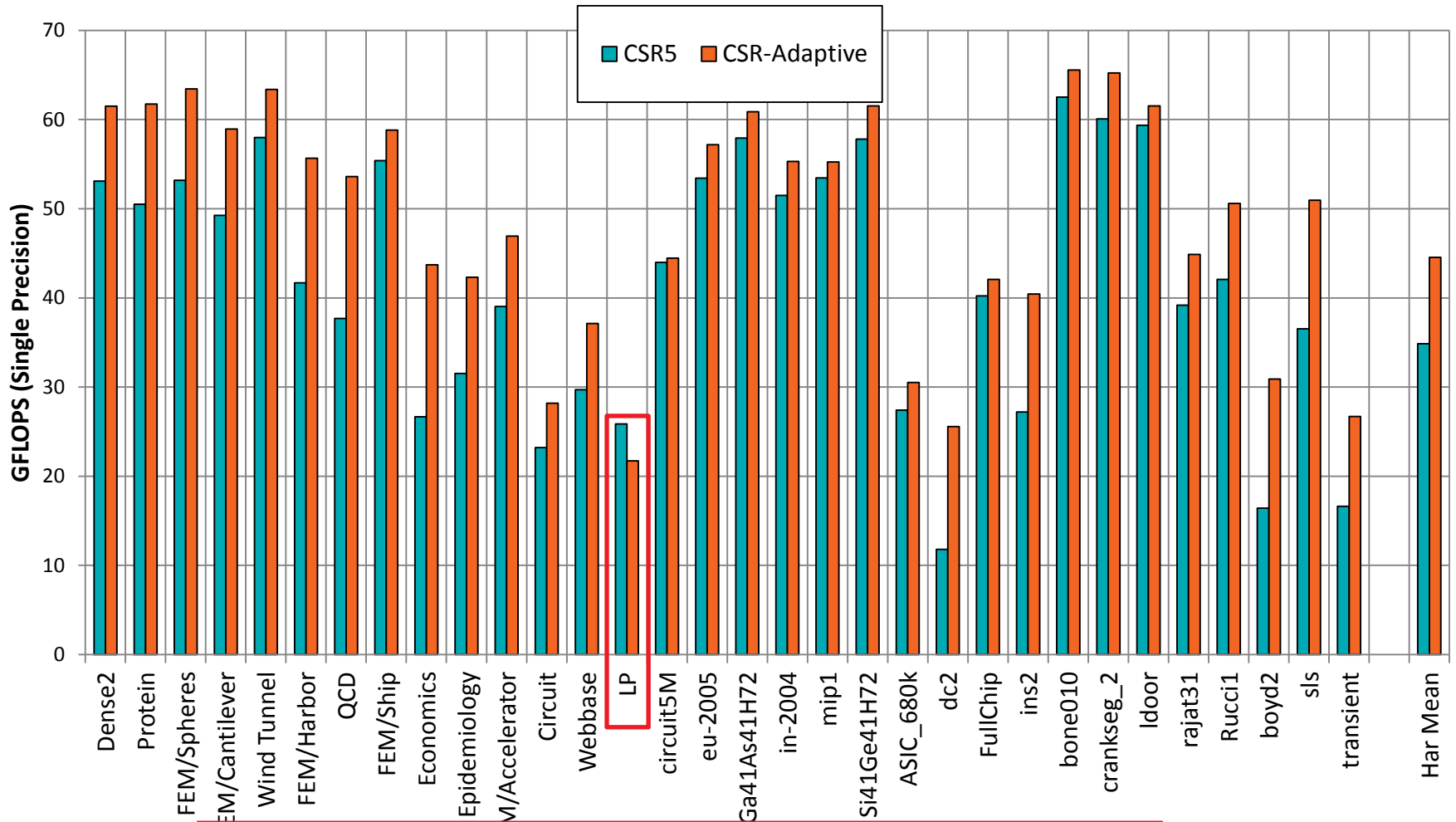
CSR-ADAPTIVE VS CSR5

AMD FIREPRO W9100 GPU (SINGLE PRECISION)



CSR-ADAPTIVE VS CSR5

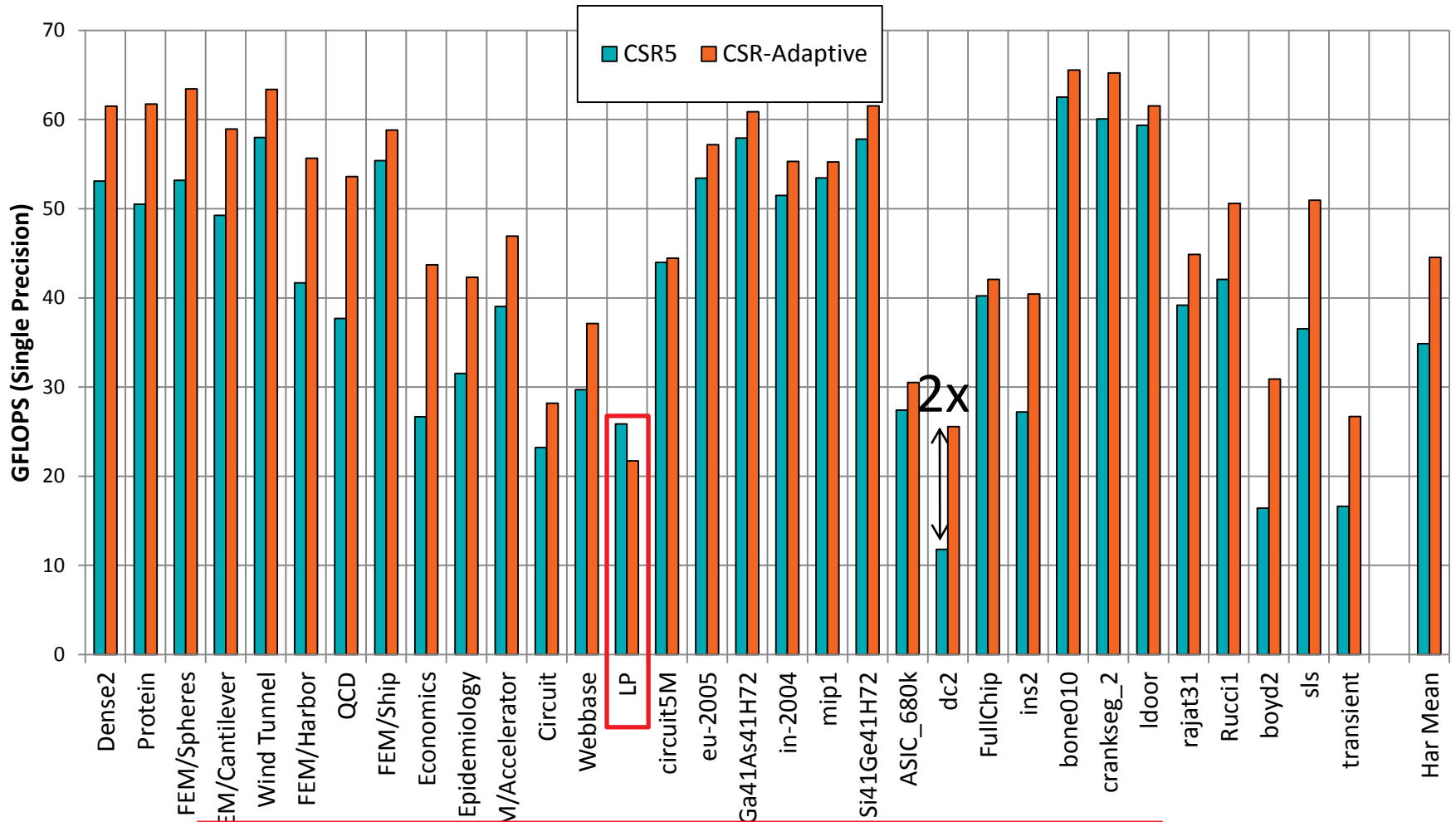
AMD FIREPRO W9100 GPU (SINGLE PRECISION)



Lose out on 1/32 matrices

CSR-ADAPTIVE VS CSR5

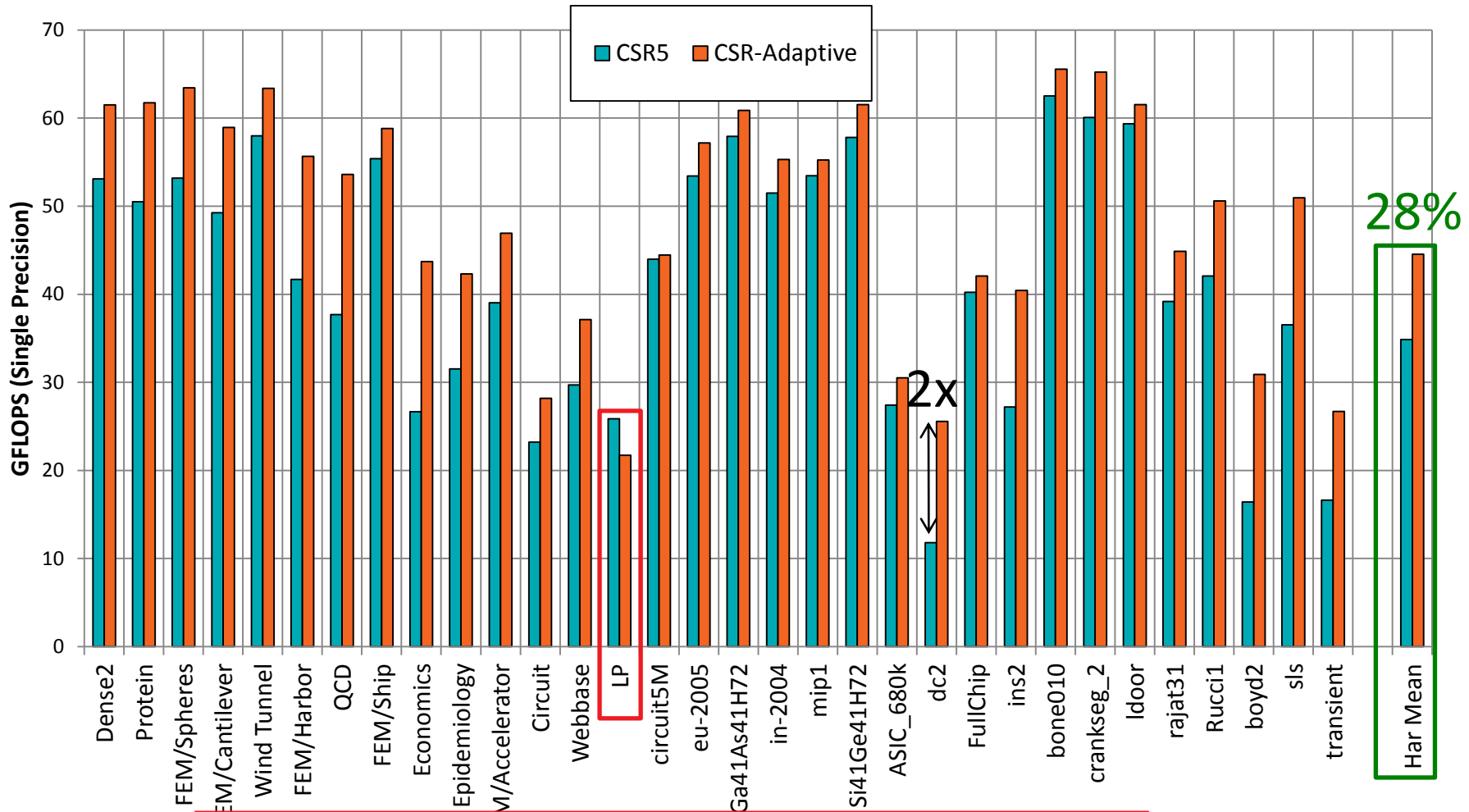
AMD FIREPRO W9100 GPU (SINGLE PRECISION)



Lose out on 1/32 matrices

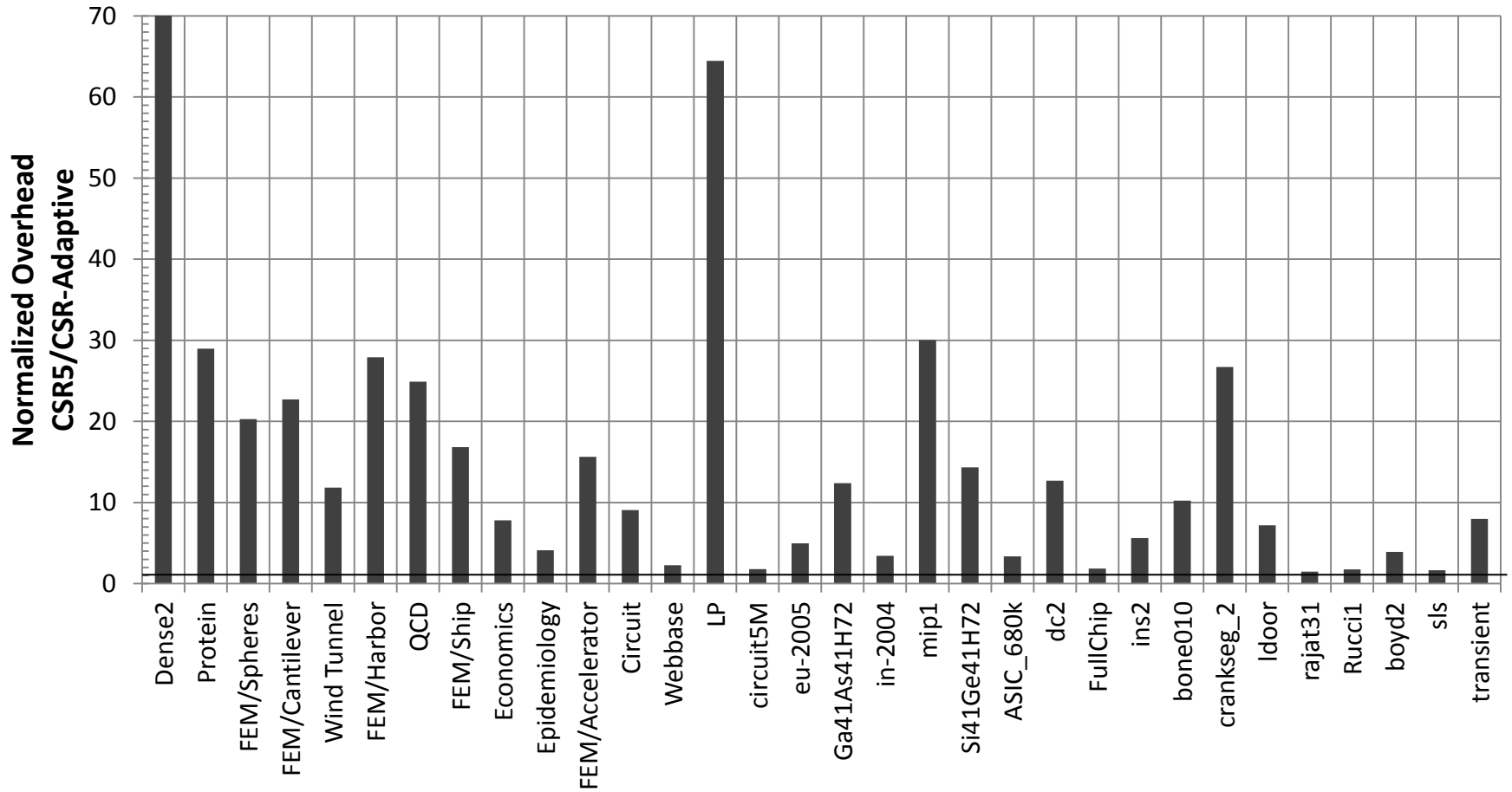
CSR-ADAPTIVE VS CSR5

AMD FIREPRO W9100 GPU (SINGLE PRECISION)

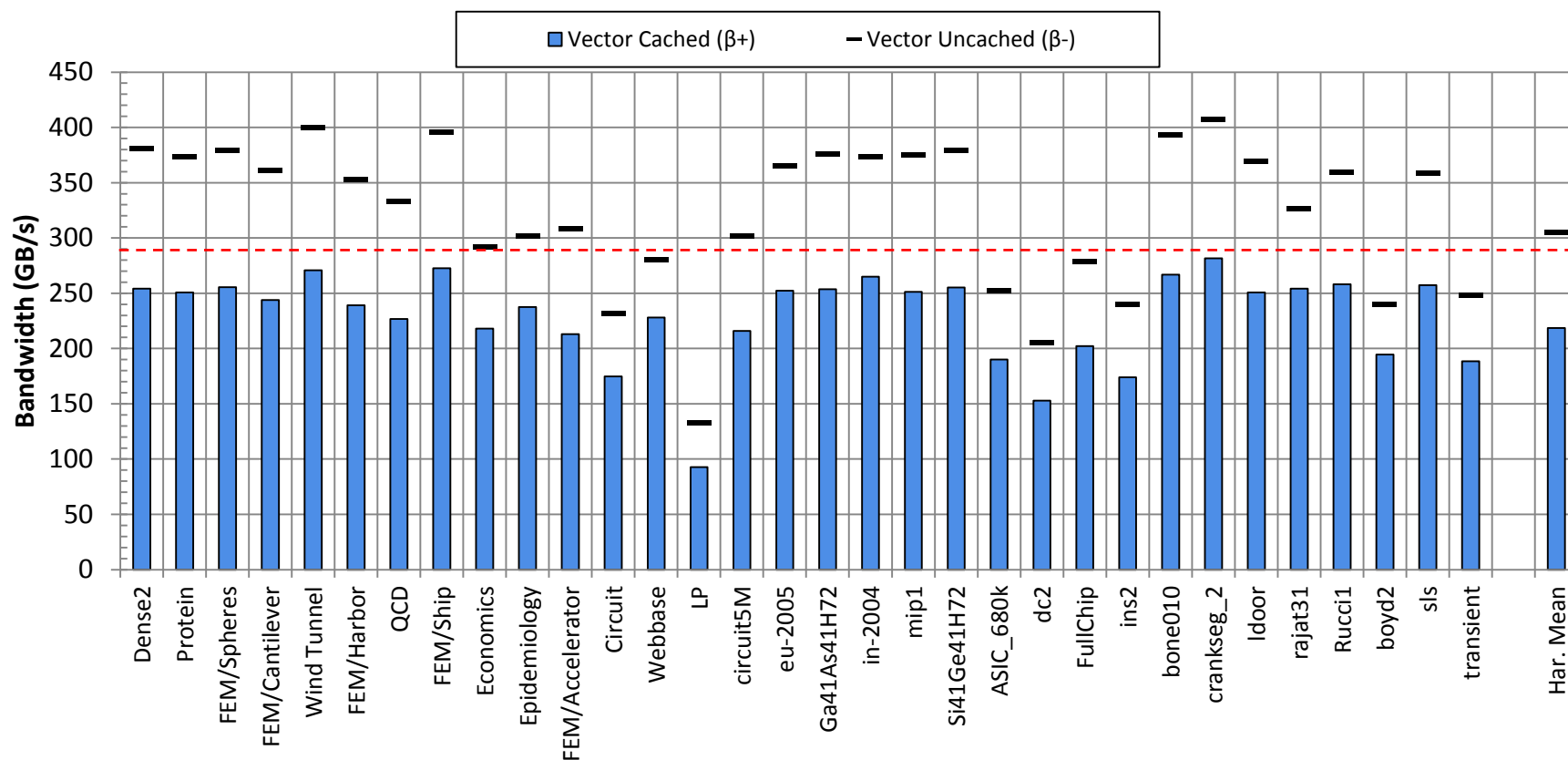


Lose out on 1/32 matrices

OVERHEAD: CSR-ADAPTIVE VS CSR5



BANDWIDTH: CSR-ADAPTIVE



CONCLUSION



Optimized GPU-based SpMV algorithm that
beats the competition by 2x
Operates within 15% of GPU's achievable b/w

Minimal overhead for
data structure generation

Shipping now as part of AMD's
clSPARSE library
<https://github.com/clMathLibraries/clSPARSE>

DISCLAIMER & ATTRIBUTION



The information presented in this document is for informational purposes only and may contain technical inaccuracies, omissions and typographical errors.

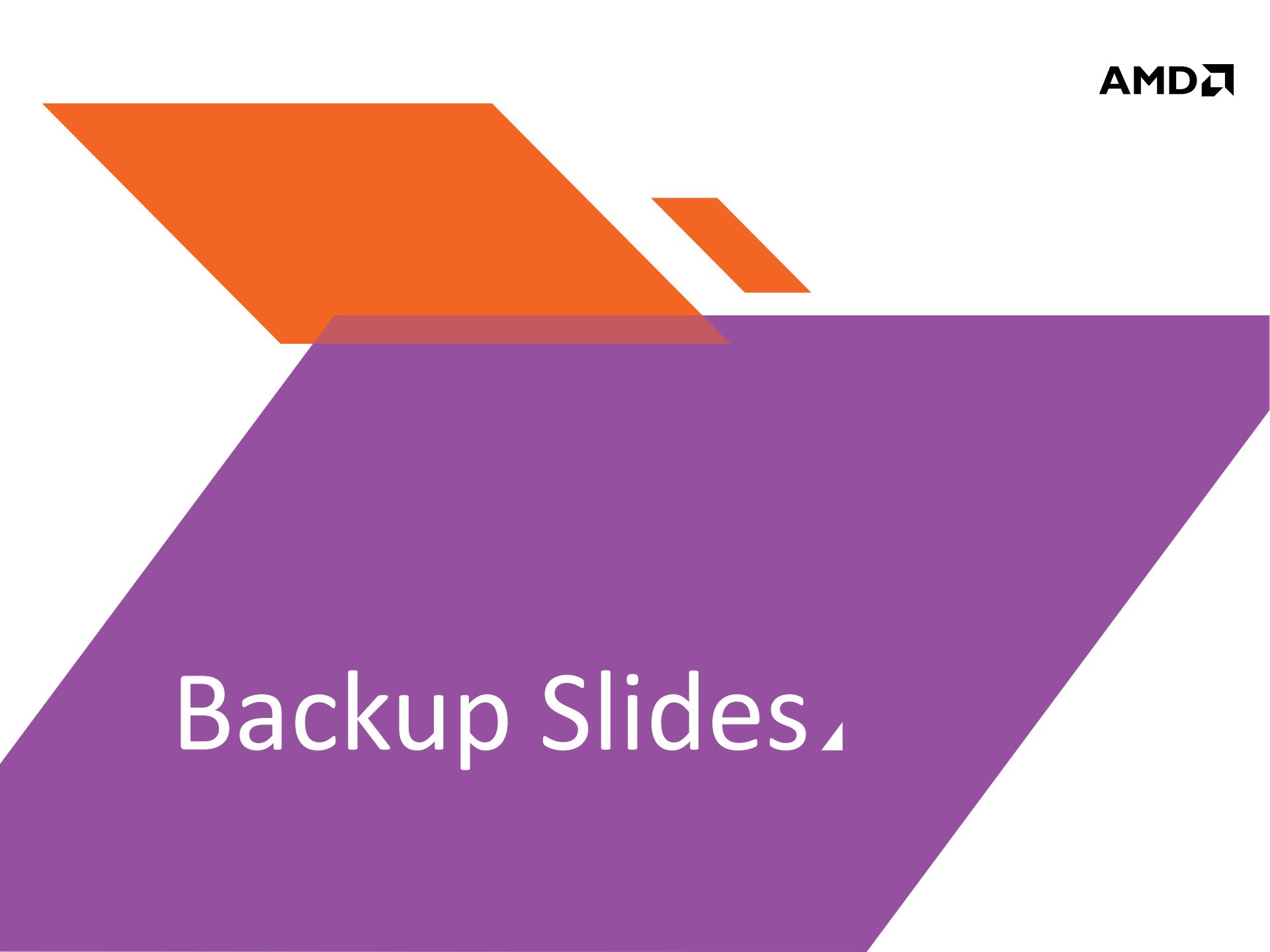
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The background features abstract geometric shapes. A large purple trapezoid occupies the lower half of the slide. Above it, there are orange shapes: a large parallelogram on the left and a smaller one on the right, both pointing towards the right. The text "Backup Slides." is centered within the purple area in a white, sans-serif font.

Backup Slides.

FINDING BLOCKS FOR CSR-ADAPTIVE

DONE ONCE ON THE CPU



row delimiters:

0	3	6	8	10	12	19	32
---	---	---	---	----	----	----	----

columns:



values:



FINDING BLOCKS FOR CSR-ADAPTIVE

DONE ONCE ON THE CPU



row delimiters:

0	3	6	8	10	12	19	32
---	---	---	---	----	----	----	----

columns:



values:



FINDING BLOCKS FOR CSR-ADAPTIVE

DONE ONCE ON THE CPU



Assuming 8 LDS entries per workgroup

row blocks:

0

row delimiters:

0	3	6	8	10	12	19	32
---	---	---	---	----	----	----	----

columns:



values:



Block 1

FINDING BLOCKS FOR CSR-ADAPTIVE

DONE ONCE ON THE CPU



Assuming 8 LDS entries per workgroup

row blocks:

0

row delimiters:

0	3	6	8	10	12	19	32
---	---	---	---	----	----	----	----

columns:



values:



Block 1

FINDING BLOCKS FOR CSR-ADAPTIVE

DONE ONCE ON THE CPU



Assuming 8 LDS entries per workgroup

row blocks:

0

row delimiters:

0	3	6	8	10	12	19	32
---	---	---	---	----	----	----	----

columns:



values:



Block 1

FINDING BLOCKS FOR CSR-ADAPTIVE

DONE ONCE ON THE CPU



Assuming 8 LDS entries per workgroup

row blocks:

0

row delimiters:

0	3	6	8	10	12	19	32
---	---	---	---	----	----	----	----

columns:



values:



Block 1

FINDING BLOCKS FOR CSR-ADAPTIVE

DONE ONCE ON THE CPU



Assuming 8 LDS entries per workgroup

row blocks:

0	3
---	---

row delimiters:

0	3	6	8	10	12	19	32
---	---	---	---	----	----	----	----

columns:



values:



Block 1

Block 2

FINDING BLOCKS FOR CSR-ADAPTIVE

DONE ONCE ON THE CPU



Assuming 8 LDS entries per workgroup

row blocks:

0	3	5
---	---	---

row delimiters:

0	3	6	8	10	12	19	32
---	---	---	---	----	----	----	----

columns:



values:



Block 1

Block 2

Block 3

FINDING BLOCKS FOR CSR-ADAPTIVE

DONE ONCE ON THE CPU



Assuming 8 LDS entries per workgroup

row blocks:

0	3	5	6	6	7
---	---	---	---	---	---

row delimiters:

0	3	6	8	10	12	19	32
---	---	---	---	----	----	----	----

columns:



values:



Block 1

Block 2

Block 3

Block 4

Block 5

FINDING BLOCKS FOR CSR-ADAPTIVE

DONE ONCE ON THE CPU



Assuming 8 LDS entries per workgroup

row blocks:

0	3	5	6	6	7
---	---	---	---	---	---

CSR Structure
Unchanged

row delimiters:

0	3	6	8	10	12	19	32
---	---	---	---	----	----	----	----

columns:



values:



Block 1

Block 2

Block 3

Block 4

Block 5

A DECENT SPMV FOR THE CPU IS SIMPLE



```
1. for (i = 0; i < nRows; i++) {  
2.   temp = 0;  
3.   for (j = rowDs[i]; j < rowDs[i+1]; j++) {  
4.     temp += vals[j] * vec[cols[j]];  
5.   }  
6.   output[i] = temp;  
7. }
```

A DECENT SPMV FOR THE CPU IS SIMPLE



#pragma omp parallel for

1. **for** (i = 0; i < nRows; i++) {
2. temp = 0;
3. **for** (j = rowDs[i]; j < rowDs[i+1]; j++) {
4. temp += vals[j] * vec[cols[j]];
5. }
6. output[i] = temp;
7. }

A DECENT SPMV FOR THE CPU IS SIMPLE



#pragma omp parallel for

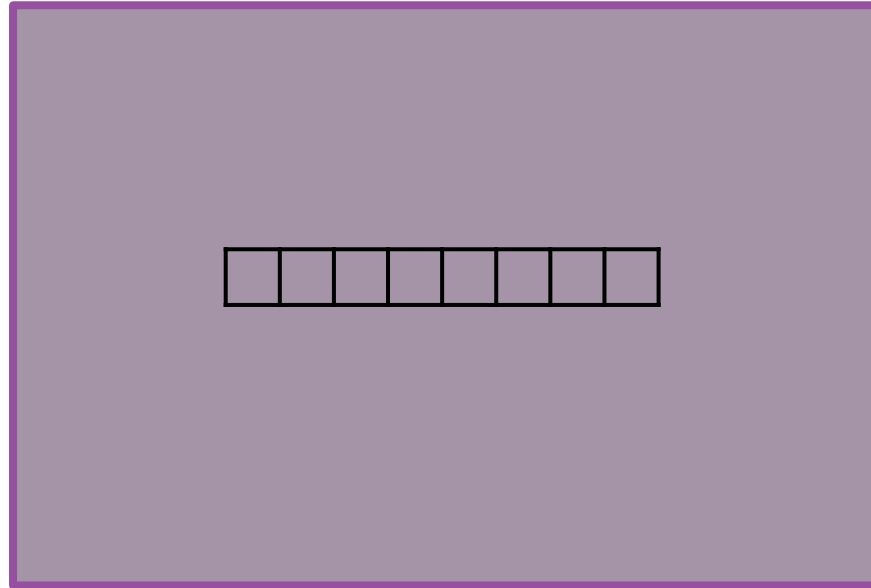
1. **for** (i = 0; i < nRows; i++) {
2. temp = 0;
3. **for** (j = rowDs[i]; j < rowDs[i+1]; j++) {
4. temp += vals[j] * vec[cols[j]];
5. }
6. output[i] = temp;
7. }

Only **seven** lines of code provides decent
SpMV performance on the CPU

CSR-VECTOR USING THE LDS



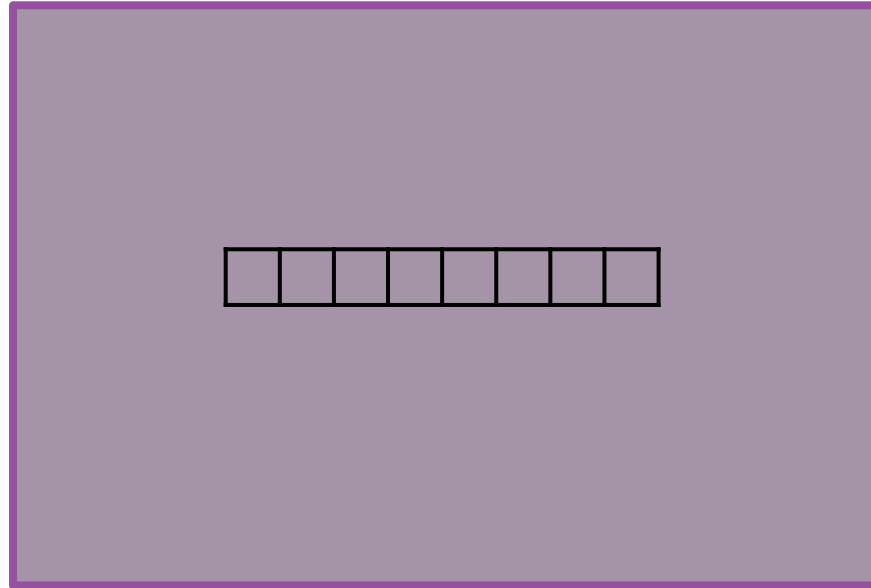
Local Data Share



CSR-VECTOR USING THE LDS



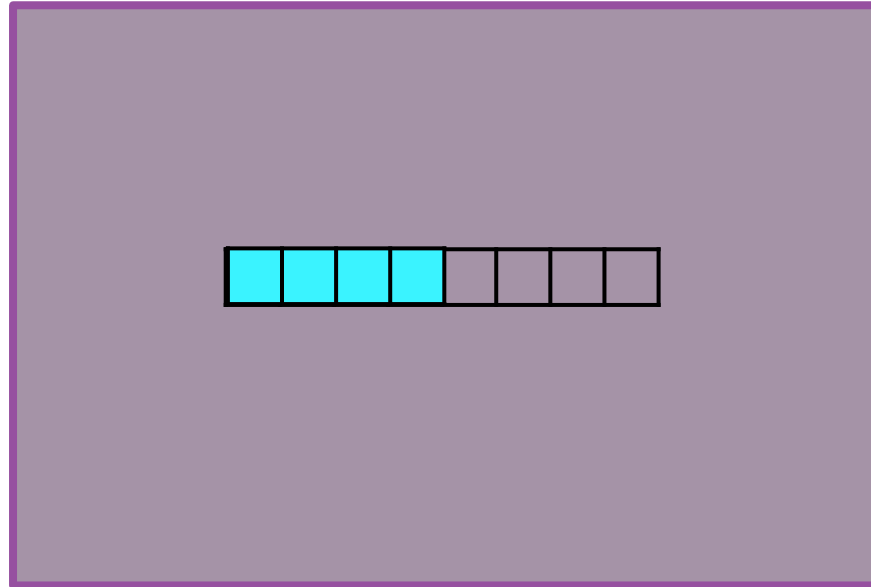
Local Data Share



CSR-VECTOR USING THE LDS



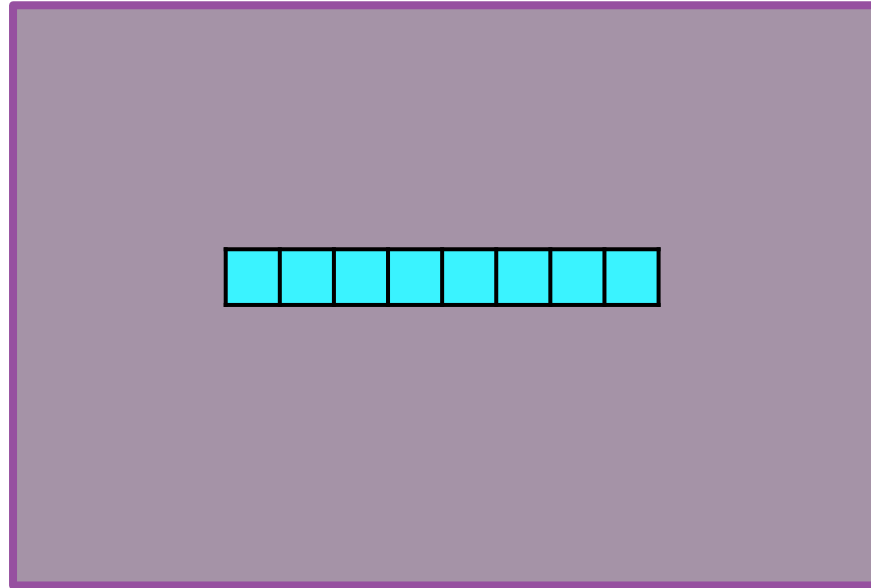
Local Data Share



CSR-VECTOR USING THE LDS



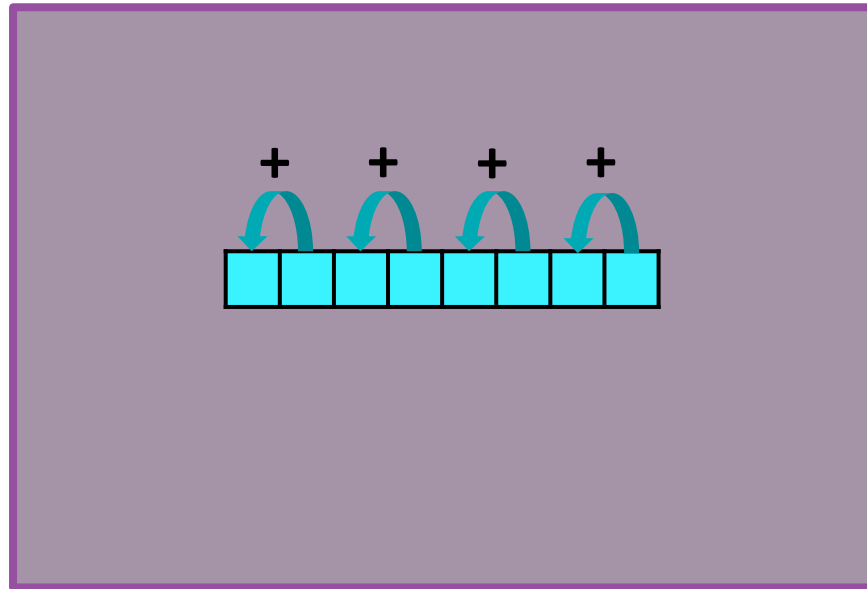
Local Data Share



CSR-VECTOR USING THE LDS



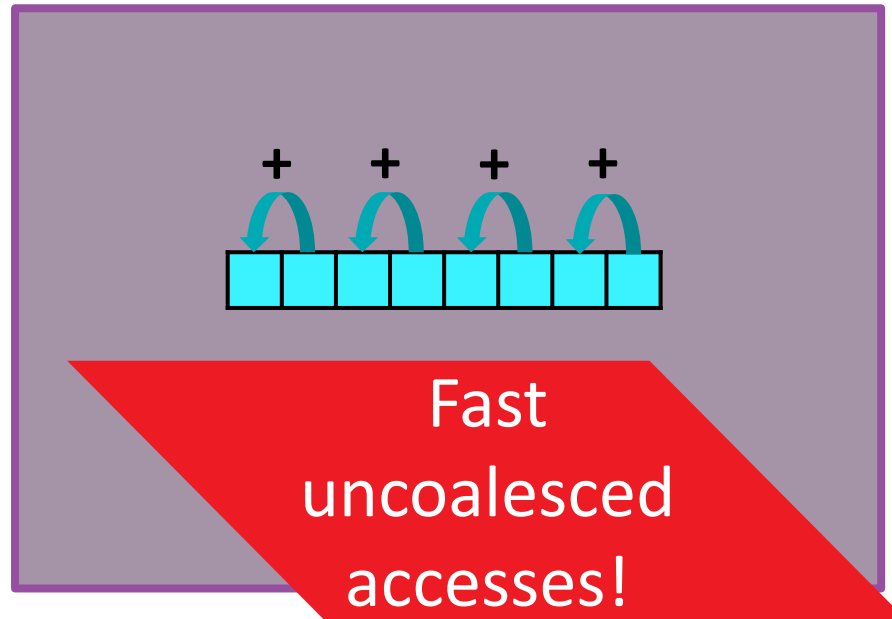
Local Data Share



CSR-VECTOR USING THE LDS



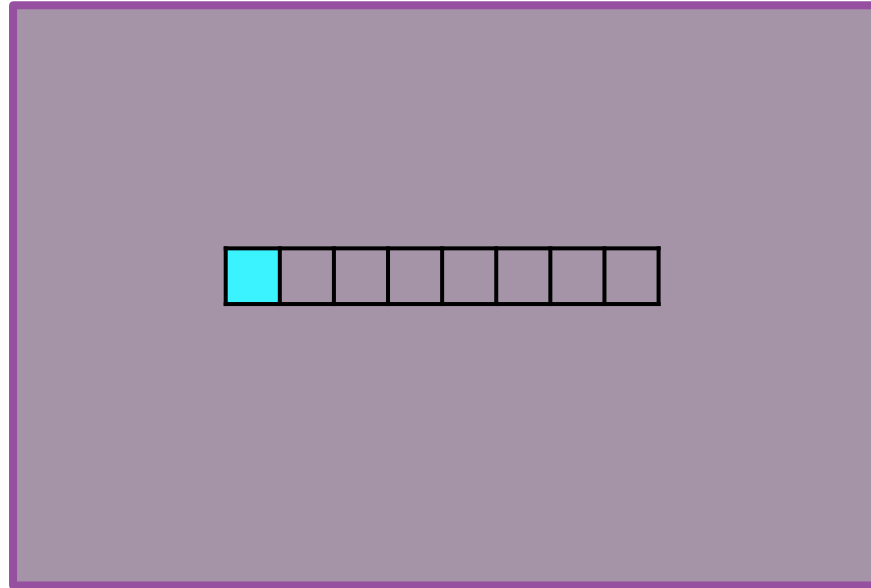
Local Data Share



CSR-VECTOR USING THE LDS



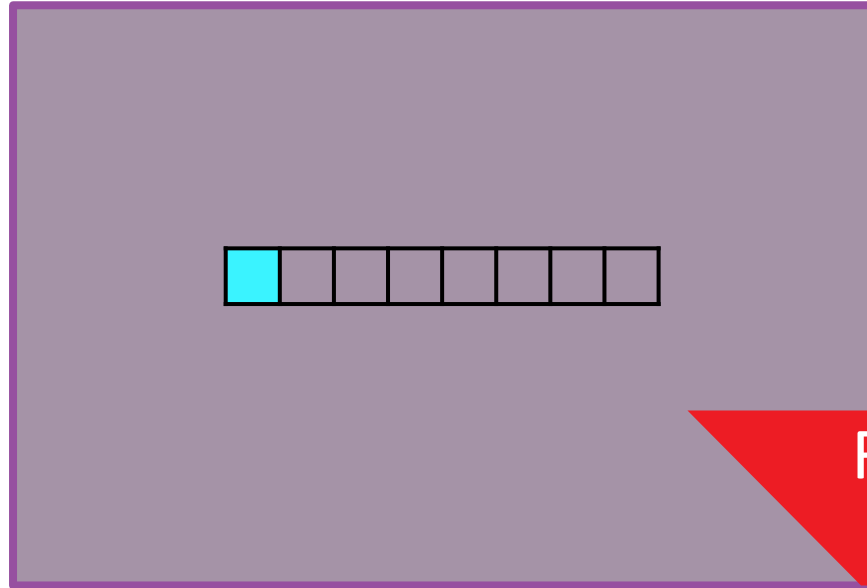
Local Data Share



CSR-VECTOR USING THE LDS



Local Data Share



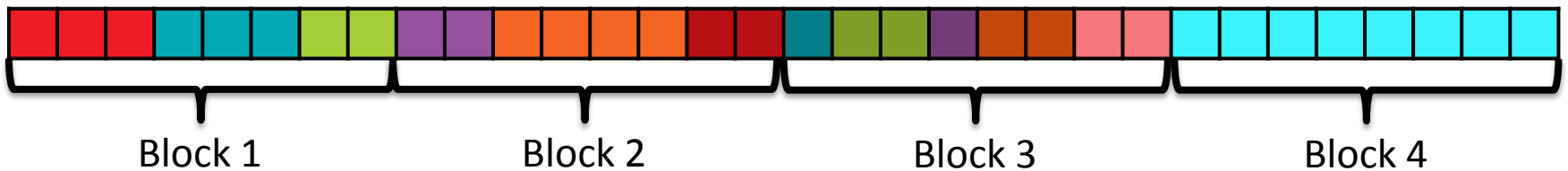
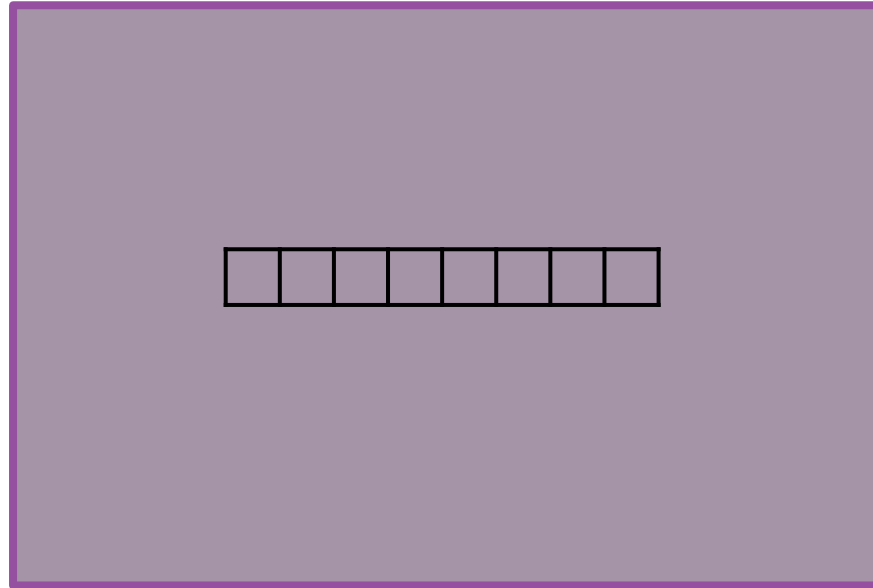
Fast accesses
everywhere



CSR-VECTOR USING THE LDS



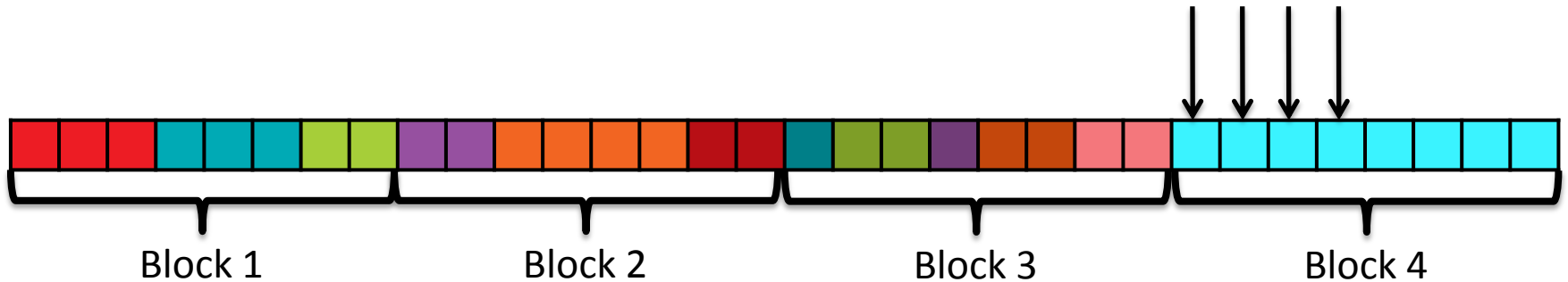
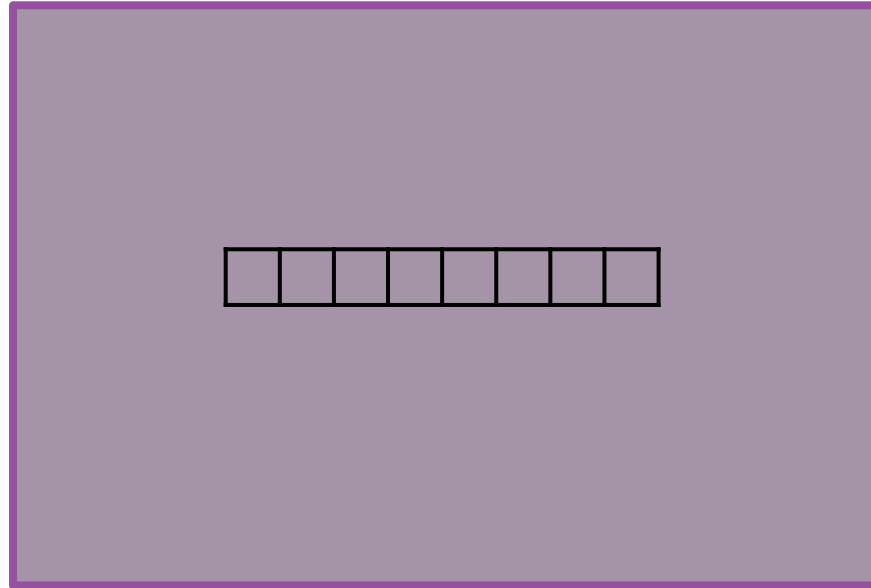
Local Data Share



CSR-VECTOR USING THE LDS



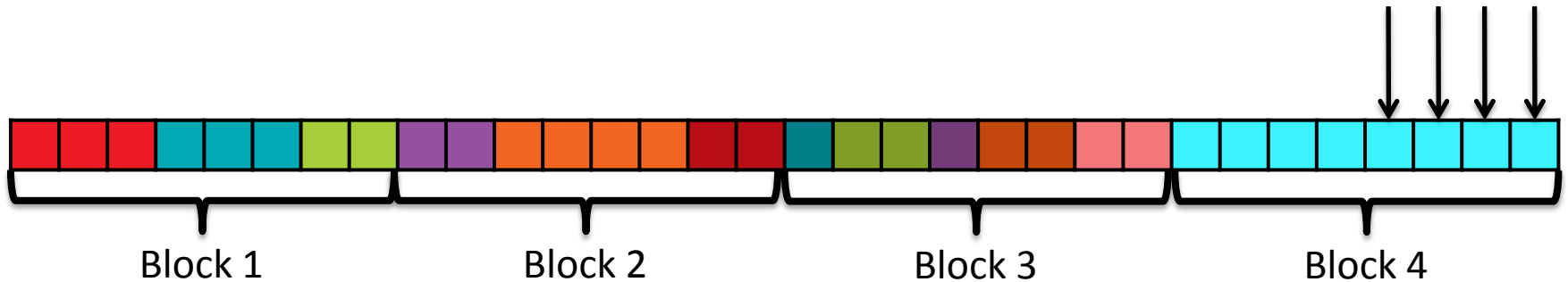
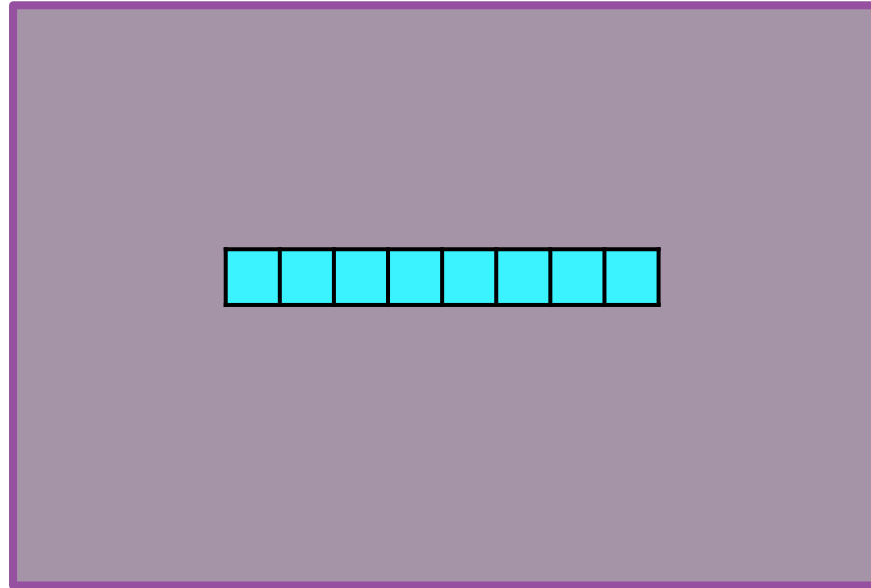
Local Data Share



CSR-VECTOR USING THE LDS



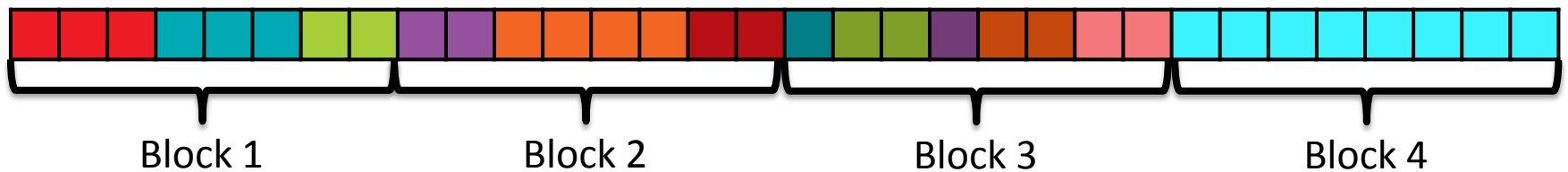
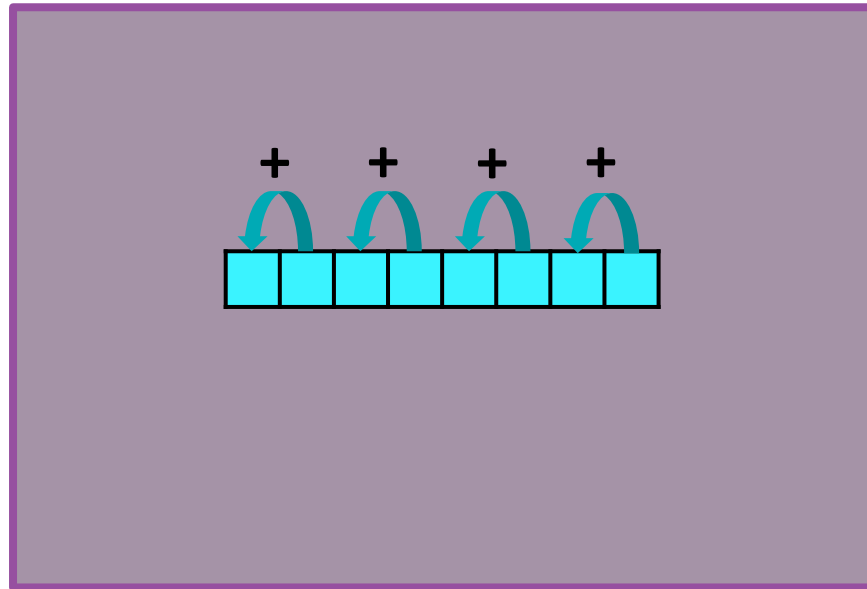
Local Data Share



CSR-VECTOR USING THE LDS



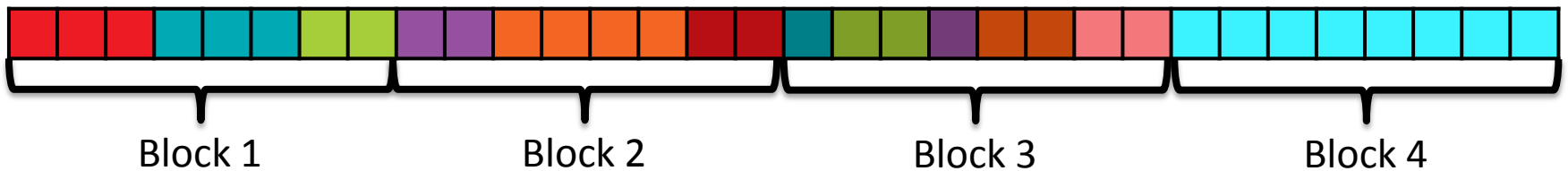
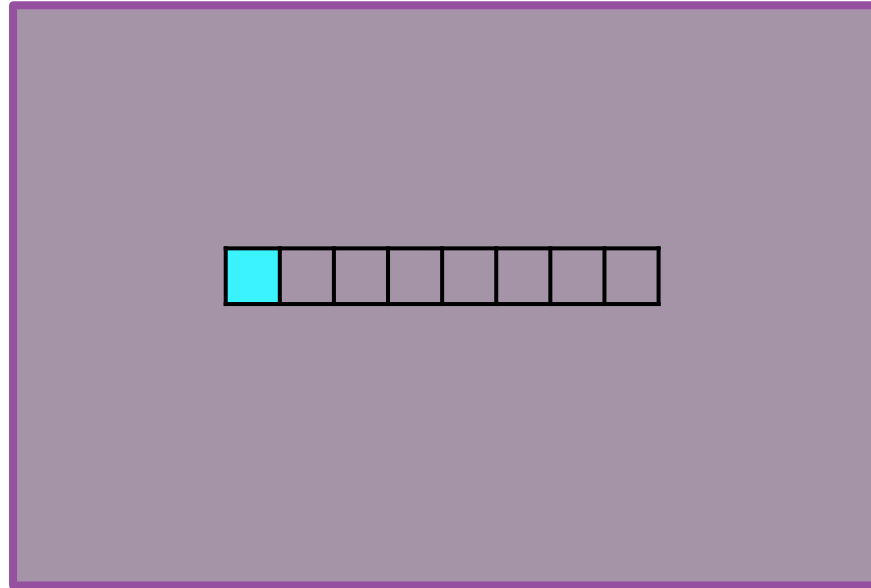
Local Data Share



CSR-VECTOR USING THE LDS



Local Data Share



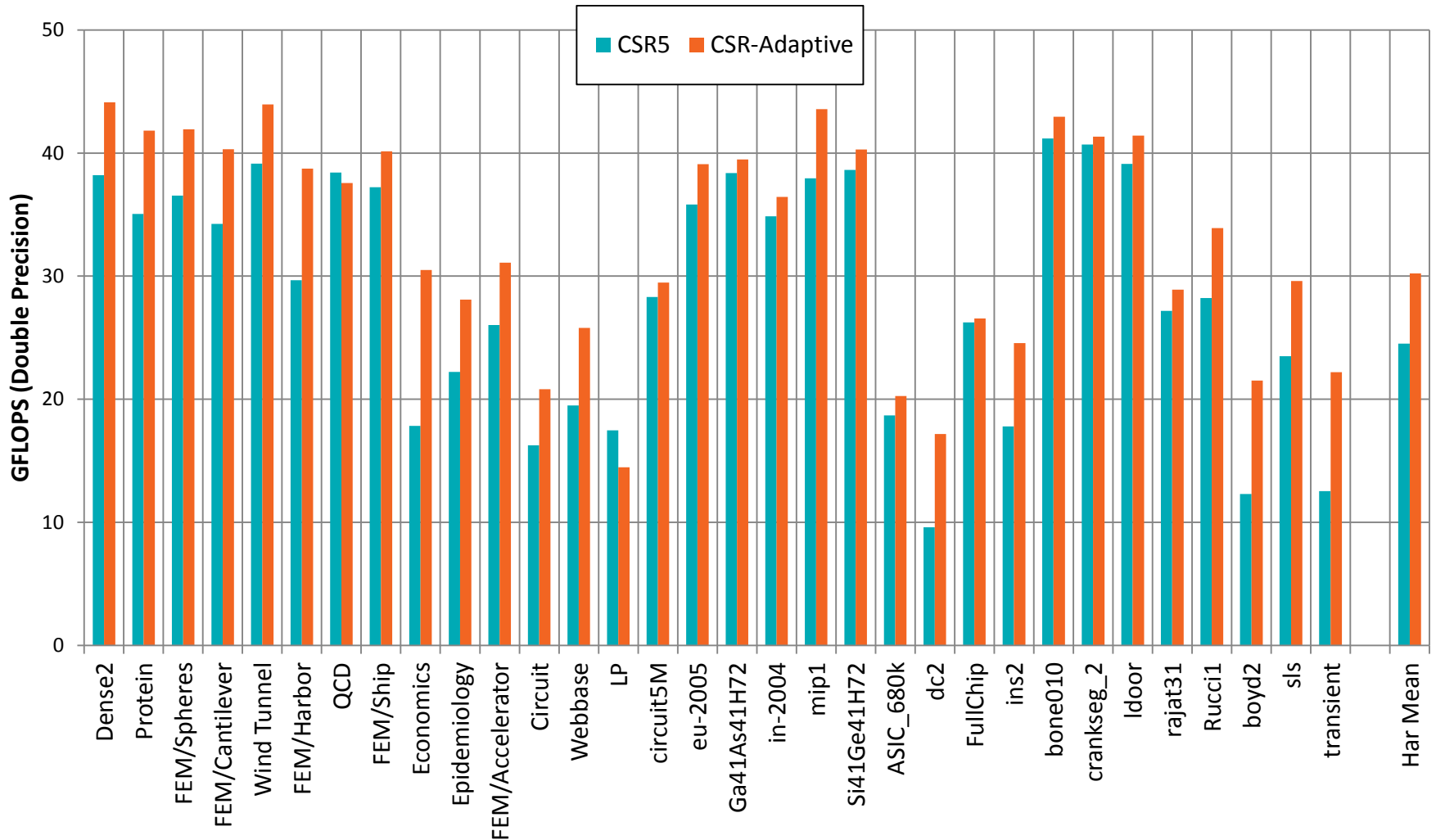
CSR-ADAPTIVE VS CSR5 (BEST IN ACADEMIA)

AMD FIREPRO W9100 GPU (DOUBLE PRECISION)



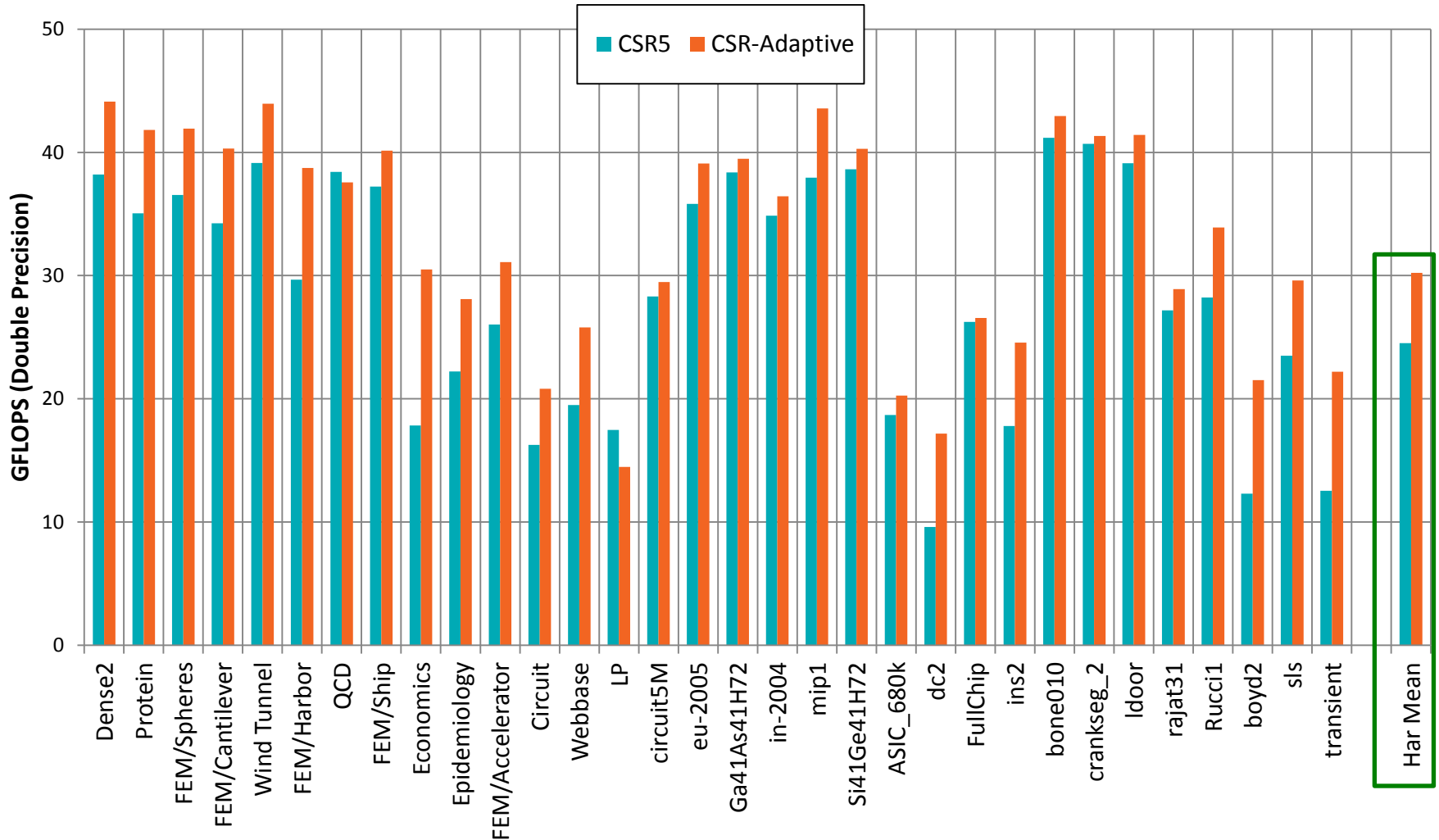
CSR-ADAPTIVE VS CSR5 (BEST IN ACADEMIA)

AMD FIREPRO W9100 GPU (DOUBLE PRECISION)

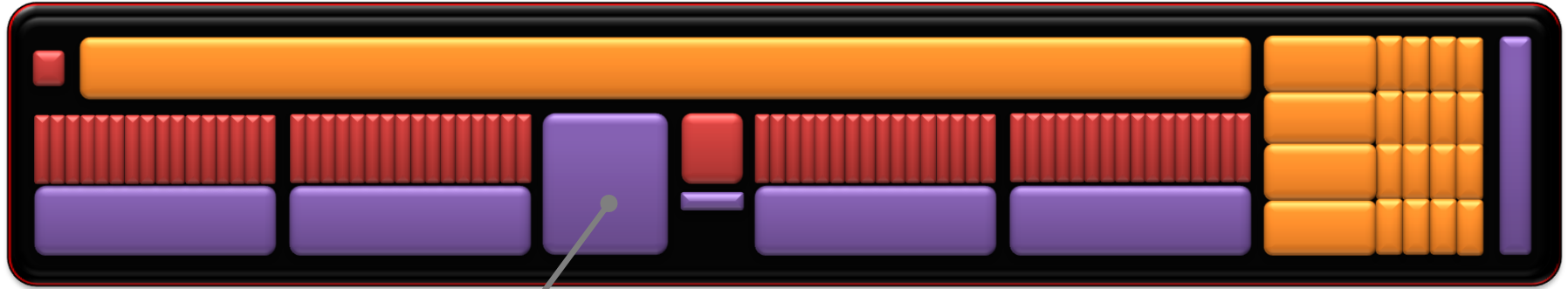


CSR-ADAPTIVE VS CSR5 (BEST IN ACADEMIA)

AMD FIREPRO W9100 GPU (DOUBLE PRECISION)



LOCAL DATA SHARE (LDS) MEMORY



**Local Data Share
(64KB)**

- ▲ Scratchpad (software-addressed) on-chip cache
- ▲ Statically divided between all workgroups in a CU
- ▲ Highly ported to allow scatter/gather (uncoalesced) accesses